Problem Set #5

This problem set is due at the start of class on Thursday, May 3rd.

1. There is a single object for sale and there are two potential buyers. Let δ be a real number such that $0 \leq \delta < 1$. The value assigned by Buyer 1 to the object is uniformly distributed over the interval $[0, 1+\delta]$. The value assigned by Buyer 2 to the object is uniformly distributed over the interval $[0, 1-\delta]$. The two values are independently distributed. Remark: The value of δ is known to the seller.

   (a) Suppose the seller decides to sell the object using a second-price auction with a reserve price r. As a function of δ, what is the optimal value of r (i.e., the value of r that maximizes the expected seller revenue)?

   (b) As a function of δ, what is the expected seller revenue of the auction associated with part (a)?

   (c) What is an optimal auction (i.e., an auction that maximizes the expected seller revenue) for the seller to use in this scenario?

   (d) As a function of δ, what is the expected seller revenue of the optimal auction of part (c)?

2. Consider a combinatorial auction with m items numbered from 1 to m. Assume that each bidder is single-minded (i.e., is only interested in acquiring a specific bundle of items) and desires an interval of consecutively numbered items. Prove that an efficient allocation (i.e., an allocation that maximizes the social welfare) can be determined in polynomial time. Hint: Use dynamic programming. CLARIFICATION ADDED 4/19/12: You should assume that you are given, for each bidder i, the bundle of items S_i that bidder i is interested in, along with the nonnegative value v_i that bidder i assigns to S_i.

3. Consider a combinatorial auction for m items with n bidders, where a bid is a vector of length $2^m - 1$ specifying a nonnegative value for each nonempty subset of the m items (the value of the empty set is assumed to be zero). Prove that an efficient allocation can be computed in time that is polynomial in the input length $n(2^m - 1)$. Hint: Use dynamic programming. Remark: The result of this question implies that when $m = O(\log n)$ an efficient allocation can be computed in time polynomial in n.

4. In a procurement auction with single-minded bidders, a single buyer needs to buy a set S of m items from n possible suppliers. Each supplier i can provide a subset S_i of S at a privately known cost c_i. Assume that $\cup_{1 \leq i \leq n} S_i = S$. The buyer needs to buy (at least one of) each of the m items, and seeks to minimize the total price paid.

(a) Prove that, given the c_i's, the following greedy algorithm yields an $H(m)$-approximation to the optimal (i.e., minimum total cost) procurement, where $H(m) = \sum_{1 \le i \le m} \frac{1}{i} \sim \ln m$. The algorithm initializes a set R to the set of all m items and a set W to the empty set. While the set R is nonempty, the algorithm determines a supplier index i minimizing the ratio $\frac{c_i}{|R \cap S_i|}$, and then updates W to $W + i$ and R to $R \setminus S_i$. Upon termination, the set W contains the supplier indices corresponding to a procurement of cost $\sum_{i \in W} c_i$.

(b) The algorithm of part (a) assumes that the c_i's are given. In a procurement auction, each supplier i submits a bid b_i that may or may not be equal to c_i. Consider a procurement auction mechanism that takes the b_i's as input and uses these values instead of the c_i's to determine the set of winning suppliers W as in part (a). Describe a suitable payment scheme so that this mechanism is individually rational and incentive-compatible. Justify your answer.