CS313H: Problem Set 5

RULES: You must indicate who you worked with (at most two others). If you worked by yourself please indicate this. You MAY NOT USE THE INTERNET OR ANY OTHER SOURCE TO LOOK UP SOLUTIONS. CHEATING WILL NOT BE TOLERATED. You may use anything we discussed in class/modules and any statement *proved* in the book.

1. Let G be a graph. An *independent set* is a collection S of some of the vertices (perhaps none, perhaps all) of G such that no two vertices from S are adjacent in G. Let $\omega(G)$ be the size of the largest independent set of G. Let $\chi(G)$ be the chromatic number of G (the chromatic number is the smallest number of colors needed to validly color a graph). If G has n vertices, show $\omega(G) \cdot \chi(G) \geq n$.

2. Is it possible for each person in a group of 9 people to be friends with exactly five of the nine? Explain your answer.

3. Show that in every (not necessarily connected) graph there is a path from every vertex u of odd degree to some other vertex v ($u \neq v$) also of odd degree.

4. Show that every connected graph on n vertices has at least $n - 1$ edges. Use induction on the number of vertices. Be careful in the inductive step– if you remove just any old vertex you may disconnect the graph, and the I.H. will not apply. The handshaking theorem may be useful here.

5. Consider a grid with height $p \geq 2$ and width $q \geq 2$, so there are pq squares in the grid. A valid walk on the grid is a walk that starts on one square and subsequently moves to adjacent squares (you cannot move diagonally). Define a *tour* to be a valid walk on the grid that touches each and every square exactly oonce and begins and ends on the same square. Prove that if p and q are odd, there does not exist a tour. (Hint: do not use induction; model the problem using a graph).