1. Let $G$ be a graph. An independent set is a collection $S$ of some of the vertices (perhaps none, perhaps all) of $G$ such that no two vertices from $S$ are adjacent in $G$. Let $\omega(G)$ be the size of the largest independent set of $G$. Let $\chi(G)$ be the chromatic number of $G$ (the chromatic number is the smallest number of colors needed to validly color a graph). If $G$ has $n$ vertices, show $\omega(G) \cdot \chi(G) \geq n$.

2. Is it possible for each person in a group of 9 people to be friends with exactly five of the nine? Explain your answer.

3. Show that in every (not necessarily connected) graph there is a path from every vertex $u$ of odd degree to some other vertex $v$ ($u \neq v$) also of odd degree.

4. Show that every connected graph on $n$ vertices has at least $n - 1$ edges. Use induction on the number of vertices. Be careful in the inductive step-- if you remove just any old vertex you may disconnect the graph, and the I.H. will not apply. The handshaking theorem may be useful here.

5. Consider a grid with height $p \geq 2$ and width $q \geq 2$, so there are $pq$ squares in the grid. A valid walk on the grid is a walk that starts on one square and subsequently moves to adjacent squares (you cannot move diagonally). Define a tour to be a valid walk on the grid that touches each and every square exactly once and begins and ends on the same square. Prove that if $p$ and $q$ are odd, there does not exist a tour. (Hint: do not use induction; model the problem using a graph).