CS311H

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Good Morning, Colleagues

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Are there any questions?

Logistics

- Extra big-O problems on piazza

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- Midterm stats:

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- Midterm stats:
  - Mean: 46 (out of 60)
  - Median: 49.2
  - High score: 64

Questions

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- Examples of usage, coming up with recurrences

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- Can d be 0?
- What if c is 0? — Theorem 1 on p. 530
- log:remember that $\log_b x = \frac{\log_d x}{\log_d b}$
- Examples of usage, coming up with recurrences
- Why is Master theorem true? (intuition, proof)

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  3. $f(x) = x^2 + x + 1$
  4. $f(x) = 5 \log x$
  5. $f(x) = floor(x)$
  6. $f(x) = ceiling(x/2)$

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Quiz

- Prove if $f(x)$ is $O(g(x))$, then $g(x)$ is $\Omega(f(x))$

Proving Master Theorem

f increasing function with $f(n) = af(n/b) + cn^d$, $a \geq 1, b \in \mathbb{N}, c, d > 0$.

- Show that if $a = b^d$ and n a power of b, then $f(n) = f(1)n^d + cn^d \log_b n$.

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- Show that if $a = b^d$ and n a power of b, then $f(n) = f(1)n^d + cn^d \log_b n$.
- Show that if $a = b^d$, then $f(n)$ is $O(n^d \log n)$.

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   Answer: $7\log n = O(n) \Rightarrow 7\log n \leq Cn$

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   So, $T(n) = 8T(n/2) + 6n + 7\log n \leq 8T(n/2) + 6n + Cn$

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   So, $T(n) = 8T(n/2) + 6n + 7\log n \leq 8T(n/2) + 6n + Cn = 8T(n/2) + (6+C)n$

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3. $T(n) = 8T(n/2) + 6n + 7\log n$
   Answer: $7\log n = O(n) \Rightarrow 7\log n \leq Cn$
   So, $T(n) = 8T(n/2) + 6n + 7\log n \leq 8T(n/2) + 6n + Cn = 8T(n/2) + (6 + C)n$
   Use Master Theorem: $8 > 2^1 \Rightarrow T(n) = O(n^{\log_2 8}) = O(n^3)$