CS311H

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Good Morning, Colleagues

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Are there any questions?

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- Thursday: wrap up and test review

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- Some sets must be undecidable:countably many programs, uncountably many sets

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- Let HI = Print "Hello"; halt;
- P $\in$ HELLO iff EQUAL(P,HI) = yes
- So EQUAL would give us a decision procedure for HELLO

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- Why not?

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- Whole topic: "Computability Theory"

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- A matter of belief...

Undecidable Problems

- Given an initial configuration in the game of life, will it go on forever?
- Given 2 context-free grammars, are they equivalent?
- Given a multi-variate polynomial ove rthe integers, does it have a root?
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  - n even: $\rightarrow$ n/2
  - n odd: $\rightarrow$ 3n+1