CS311H

Prof: Peter Stone

Department of Computer Science
The University of Texas at Austin

Good Morning, Colleagues

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Are there any questions?

Logistics

- Final: Sat., Dec. 14, 7pm-10pm, JGB 2.216

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  - Think about what you've learned…

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Answer: Under rational domain, the predicate is false because for all x, y where $x < y$ there always exists $z = \frac{x+y}{2}$ which satisfies that condition that $x < z < y$. So for all x, such y doesn't exist. which means the predicate is false.

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Under integer domain, there exists $y = x + 1$ such that no integer z exists such that $x < z < y$. Thus the predicate is true.

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  Use this fact to prove: "There are infinitely many primes"

Solution

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7. Then m is divisible by q.
8. Since m is not divisible by any p_i, prime q is not equal to any of p_i. Contradiction.

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Every prime has a pre-image (surjective)

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- Proof: uses Eurlerian circuits

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- 360 - 5! = 360 - 120 = 240

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When $a \leq 1$, we have $C = 1, k = 1$.

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$$n! \;=\; n \times (n-1) ... \frac{n}{2} \times (\frac{n}{2} - 1) ... \times 1$$

Proof (cont.)

When $a > 1$, let $k = 2a^2$, when $n > k$ we have $\frac{n}{2} > a^2$ and

$$\begin{aligned} n! &= n \times (n-1)...\frac{n}{2} \times (\frac{n}{2} - 1)... \times 1 \\ &> \underbrace{a^2 \times a^2 ... a^2}_{\frac{n}{2}} \times (\frac{n}{2} - 1)... \times 1 \end{aligned}$$

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Thus we have $C = 1$, $k = \max(1, 2a^2)$ such that for all $x > k$, $a^n < Cn!$. So we have $a^n = O(n!)$. Proof completed.

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Then we prove f is surjective then f is injective. Assume BWOC f is not injective which means there exists x, y such that $f(x) = f(y) = z$. Thus we have $|B| \leq |A - \{x, y\}| + 1 = |A| - 2 + 1 = |A| - 1$

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Then we prove f is surjective then f is injective. Assume BWOC f is not injective which means there exists x, y such that $f(x) = f(y) = z$. Thus we have $|B| \leq |A - \{x, y\}| + 1 = |A| - 2 + 1 = |A| - 1$ which means f is not surjective.

Functions

- Let A be a finite set and $f : A \to A$ be a function. Prove that f is injective if and only if f is surjective.

Proof: First prove that if f is injective then f is surjective. Let B be the set of the images of $f(x)$. Since f is injective, we have $|B| = |A|$. Since we have $B \subseteq A$ and A has finite number of elements, we have $B = A$ which means f is surjective.
Then we prove f is surjective then f is injective. Assume BWOC f is not injective which means there exists x, y such that $f(x) = f(y) = z$. Thus we have $|B| \leq |A - \{x, y\}| + 1 = |A| - 2 + 1 = |A| - 1$ which means f is not surjective. Contradiction.

Other problem types

- DeMorgan's laws and other propositional logic
- Induction
- Planar graphs
- Graph coloring
- Recurrences
- Master theorem
- Proving program correctness
- Undecidability

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- I've really enjoyed teaching you

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- **Thank you** for your contributions to the class...

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- And good luck in your future CS courses!

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- See you Dec. 14th