Good Morning, Colleagues
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Are there any questions?
Logistics

- Final: Sat., Dec. 14, 7pm-10pm, JGB 2.216
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  – Covers the whole class
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  - Difficulty like the midterms (but longer)
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- How to study
Logistics

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  - Review modules, slides, notes, book
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- Office hour Monday: 1:00-2:00
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- Please complete the official survey
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- Please complete the official survey
  - Think about what you’ve learned...
Course Recap

- Propositional logic and Satisfiability
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- Predicates, and Quantifiers
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- Predicates, and Quantifiers
- Basic proof techniques,
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- (*) Infinite sets
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- Special types of graphs (planar, bipartite)
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- Big O, program efficiency,
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- Big O, program efficiency, and master theorem
- (*) Proving program correctness
- (*) Undecidability
Test Review

- Just a start to jog your memory
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• Just a start to jog your memory

• Can’t cover all problem types
Test Review

• Just a start to jog your memory
• Can’t cover all problem types
• Will go through some of these quickly
Test Review

- Just a start to jog your memory
- Can’t cover all problem types
- Will go through some of these quickly
- Continue on your own for the next 9 days!
True or False?

- Predicate: \( \forall x \exists y (x < y \land \neg \exists z (x < z \land z < y)) \)
  - Domain: rational numbers
True or False?

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  - Domain: integers
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  - Domain: rational numbers
  - Domain: integers

Answer: Under rational domain, the predicate is false because for all $x, y$ where $x < y$ there always exists $z = \frac{x+y}{2}$ which satisfies that condition that $x < z < y$. So for all $x$, such $y$ doesn’t exist. which means the predicate is false.
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Under integer domain, there exists $y = x + 1$ such that no integer $z$ exists such that $x < z < y$. Thus the predicate is true.
How many primes are there?

- Assume the following fact:
  “Every integer larger than 1 is either prime or can be written as a product of primes”
How many primes are there?

• Assume the following fact:
  “Every integer larger than 1 is either prime or can be written as a product of primes”
Use this fact to prove: “There are infinitely many primes”
Solution

1. Suppose a finite number of primes \( n \) and seek contradiction.
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2. Let $p_1, \ldots, p_n$ be the primes, and define $m = (p_1 \times \ldots \times p_n) + 1$
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3. For every prime $p_i$, $m$ is not divisible by $p_i$ since there will be a remainder of 1.
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3. For every prime $p_i$, $m$ is not divisible by $p_i$ since there will be a remainder of 1.
4. Use the fact: $m$ is either prime or can be written as a product of primes.
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4. Use the fact: $m$ is either prime or can be written as a product of primes.
5. If $m$ is prime, it is bigger than all of $p_1, ..., p_n$, and therefore not equal to any of them. Contradiction.
Solution

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6. If $m$ is not prime, it is a product of primes. Let $q$ be one of these primes.
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7. Then $m$ is divisible by $q$. 
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2. Let \( p_1, \ldots, p_n \) be the primes, and define \( m = (p_1 \times \ldots \times p_n) + 1 \).

3. For every prime \( p_i \), \( m \) is not divisible by \( p_i \) since there will be a remainder of 1.

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5. If \( m \) is prime, it is bigger than all of \( p_1, \ldots, p_n \), and therefore not equal to any of them. Contradiction.

6. If \( m \) is not prime, it is a product of primes. Let \( q \) be one of these primes.

7. Then \( m \) is divisible by \( q \).

8. Since \( m \) is not divisible by any \( p_i \), prime \( q \) is not equal to any of \( p_i \). Contradiction.
Infinite sets

- Prove that the cardinality of the prime numbers is the same as the cardinality of the integers by defining a bijection from the integers to the primes.
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To show:
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To show:
Every integer has a unique image (injective)
Every prime has a pre-image (surjective)
Graphs

- Prove that any bipartite graph with $t$ vertices has at most $\frac{t^2}{4}$ edges.
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Proof: If $G$ is a bipartite graph, $G$ can be partition into vertex set $A$ and $B$ such that $v(A) + v(B) = t$ and there is no edge within set $A$ and $B$. 
Prove that any bipartite graph with \( t \) vertices has at most \( \frac{t^2}{4} \) edges.

Proof: If \( G \) is a bipartite graph, \( G \) can be partition into vertex set \( A \) and \( B \) such that \( v(A) + v(B) = t \) and there is no edge within set \( A \) and \( B \). For every vertex in \( A \), its degree is at most \( v(B) \).
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Graphs

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• Prove that any bipartite graph with $t$ vertices has at most $\frac{t^2}{4}$ edges.

Proof: If $G$ is a bipartite graph, $G$ can be partition into vertex set $A$ and $B$ such that $v(A) + v(B) = t$ and there is no edge within set $A$ and $B$. For every vertex in $A$, its degree is at most $v(B)$, thus the total number of edges are at most $|E| \leq v(A)v(B) = v(A)(t - v(A)) = \frac{t^2}{4} - (v(A) - \frac{t}{2})^2 \leq \frac{t^2}{4}$

Proof completed.
Memory wheels

- Definition: a cycle of bits such that every n-bit pattern occurs among adjacent bits
Memory wheels

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• Example: memory wheel with 8 bits that contains all 3-bit patterns
Memory wheels

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• Theorem: For every $n$, a memory wheel exists of size $2^n$ which has all $n$-bit patterns
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- Theorem: For every $n$, a memory wheel exists of size $2^n$ which has all $n$-bit patterns

- Proof: uses Eulerian circuits
Counting

- How many ways are there to sit 7 people at a round table with 7 chairs?
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- Consider two ways the same if everyone has the same 2 neighbors (regardless of which side they are on)
Counting

- How many ways are there to sit 7 people at a round table with 7 chairs?
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  - What if there are 2 who can’t sit next to each other?
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- \( \frac{6!}{2} = 360 \)
Counting

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- $\frac{6!}{2} = 360$

- $360 - 5! = 360 - 120 = 240$
• Let $a$ be any positive number. Show that $a^n = O(n!)$. 
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Proof

Note we have

\[ a^n = a \times a \cdots a \]

and

\[ n! = n \times (n - 1) \cdots 2 \times 1 \]

When \( a \leq 1 \), we have \( C = 1, k = 1 \).
Proof

Note we have

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and

\[ n! = n \times (n - 1) \ldots 2 \times 1 \]

When \( a \leq 1 \), we have \( C = 1 \), \( k = 1 \).
When \( a > 1 \), \ldots
When $a > 1$, let $k = 2a^2$. 
Proof (cont.)

When $a > 1$, let $k = 2a^2$, when $n > k$ we have $\frac{n}{2} > a^2$ and
Proof (cont.)

When $a > 1$, let $k = 2a^2$, when $n > k$ we have $\frac{n}{2} > a^2$ and

$$n! = n \times (n - 1) \cdots \frac{n}{2} \times \left(\frac{n}{2} - 1\right) \cdots \times 1$$
When $a > 1$, let $k = 2a^2$, when $n > k$ we have $\frac{n}{2} > a^2$ and

\[
\begin{align*}
\text{n!} & = n \times (n-1) \cdots \frac{n}{2} \times \left(\frac{n}{2} - 1\right) \cdots \times 1 \\
& > \underbrace{a^2 \times a^2 \cdots a^2}_{\frac{n}{2}} \times \left(\frac{n}{2} - 1\right) \cdots \times 1
\end{align*}
\]
Proof (cont.)

When $a > 1$, let $k = 2a^2$, when $n > k$ we have $\frac{n}{2} > a^2$ and

$$n! = n \times (n - 1) \ldots \frac{n}{2} \times \left(\frac{n}{2} - 1\right) \ldots \times 1$$

$$> a^2 \times a^2 \ldots a^2 \times \left(\frac{n}{2} - 1\right) \ldots \times 1$$

$$> (a^2)^{\frac{n}{2}}$$
When $a > 1$, let $k = 2a^2$, when $n > k$ we have $\frac{n}{2} > a^2$ and

\[
\begin{align*}
\text{n!} & = n \times (n-1)... \frac{n}{2} \times (\frac{n}{2} - 1)... \times 1 \\
& > a^2 \times a^2 ... a^2 \times (\frac{n}{2} - 1)... \times 1 \\
& > (a^2)^{\frac{n}{2}} \\
& = a^n
\end{align*}
\]
When \( a > 1 \), let \( k = 2a^2 \), when \( n > k \) we have \( \frac{n}{2} > a^2 \) and

\[
\begin{align*}
n! &= n \times (n - 1) \cdots \frac{n}{2} \times (\frac{n}{2} - 1) \cdots \times 1 \\
&> a^2 \times a^2 \cdots a^2 \times (\frac{n}{2} - 1) \cdots \times 1 \\
&> (a^2)^{\frac{n}{2}} \\
&= a^n
\end{align*}
\]

Thus we have \( C = 1, \max(1, 2a^2) \) such that for all \( x > k \), \( a^n < Cn! \). So we have \( a^n = O(n!) \). Proof completed.
Let \( A \) be a finite set and \( f : A \to A \) be a function. Prove that \( f \) is injective if and only if \( f \) is surjective.
Functions

Let $A$ be a finite set and $f : A \rightarrow A$ be a function. Prove that $f$ is injective if and only if $f$ is surjective.

Proof: First prove that if $f$ is injective then $f$ is surjective.
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Proof: First prove that if \( f \) is injective then \( f \) is surjective. Let \( B \) be the set of the images of \( f(x) \).
• Let $A$ be a finite set and $f : A \rightarrow A$ be a function. Prove that $f$ is injective if and only if $f$ is surjective.

Proof: First prove that if $f$ is injective then $f$ is surjective. Let $B$ be the set of the images of $f(x)$. Since $f$ is injective, we have $|B| = |A|$. 
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Proof: First prove that if $f$ is injective then $f$ is surjective. Let $B$ be the set of the images of $f(x)$. Since $f$ is injective, we have $|B| = |A|$. Since we have $B \subseteq A$ and $A$ has finite number of elements, we have $B = A$ which means $f$ is surjective.
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Then we prove $f$ is surjective then $f$ is injective. Assume BWOC $f$ is not injective which means there exists $x, y$ such that $f(x) = f(y) = z$. 

Peter Stone
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- Let $A$ be a finite set and $f : A \to A$ be a function. Prove that $f$ is injective if and only if $f$ is surjective.

Proof: First prove that if $f$ is injective then $f$ is surjective. Let $B$ be the set of the images of $f(x)$. Since $f$ is injective, we have $|B| = |A|$. Since we have $B \subseteq A$ and $A$ has finite number of elements, we have $B = A$ which means $f$ is surjective.

Then we prove $f$ is surjective then $f$ is injective. Assume BWOC $f$ is not injective which means there exists $x, y$ such that $f(x) = f(y) = z$. Thus we have $|B| \leq |A - \{x, y\}| + 1 = |A| - 2 + 1 = |A| - 1$.
Functions

• Let \( A \) be a finite set and \( f : A \rightarrow A \) be a function. Prove that \( f \) is injective if and only if \( f \) is surjective.

Proof: First prove that if \( f \) is injective then \( f \) is surjective. Let \( B \) be the set of the images of \( f(x) \). Since \( f \) is injective, we have \( \lvert B \rvert = \lvert A \rvert \). Since we have \( B \subseteq A \) and \( A \) has finite number of elements, we have \( B = A \) which means \( f \) is surjective.

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Other problem types

- DeMorgan’s laws and other propositional logic
- Induction
- Planar graphs
- Graph coloring
- Recurrences
- Master theorem
- Proving program correctness
- Undecidability
Dismount

- I’ve really enjoyed teaching you
Dismount

- I’ve really enjoyed teaching you
- **Thank you** for your contributions to the class...
Dismount

• I’ve really enjoyed teaching you

• Thank you for your contributions to the class... for being good colleagues
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- Good luck on the final.
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And good luck in your future CS courses!
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- And good luck in your future CS courses!
- See you Dec. 14th