Good Morning, Colleagues
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Are there any questions?
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- How do we write “There exists one and only one” (and its negation)?
Logistics

- Office hours - try to let us know in advance if you’re coming
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- Keep posting on piazza
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- First homework **due at start of class**
Some important concepts

- Multiple ways of converting the same English sentence to logic
- s.t. = “such that”
- Dogs and collars problem
Translate these statements into English:

1. \( \forall x [(H(x) \land \neg \exists y M(x, y)) \rightarrow U(x)] \)
   where \( H(x) = "x \ is \ a \ man" \), \( M(x, y) = "x \ is \ married \ to \ y" \),
   \( U(x) = "x \ is \ unhappy" \).
Translate these statements into English:

1. \( \forall x[(H(x) \land \neg \exists y M(x, y)) \rightarrow U(x)] \)
   where \( H(x) = \text{“x is a man”} \), \( M(x, y) = \text{“x is married to y”} \),
   \( U(x) = \text{“x is unhappy”} \).

2. \( \exists z(P(z, Jake) \land S(z, Alex) \land W(Alex)) \)
   \( P(z, x) = \text{“z is a parent of x”} \), \( S(z, y) = \text{“z and y are siblings”} \),
   \( W(y) = \text{“y is a woman”} \).

3. \( \forall n((P(n) \land n > 2) \rightarrow \neg \exists a, b, c (P(a) \land P(b) \land P(c) \land (a^n + b^n = c^n))) \)
   where \( P(n) = \text{“n is a positive integer”} \).
Translate the following statements into logical notation

No new predicates (just use common mathematical symbols), where the domain is natural numbers.

1. \(x\) is a perfect square.

2. \(x\) is a multiple of \(y\).

3. \(p\) is prime.
Translate the following statements into logical notation

No new predicates (just use common mathematical symbols), where the domain is natural numbers.

1. $x$ is a perfect square.
   $\exists y (x = y^2)$

2. $x$ is a multiple of $y$.
   $\exists z (x = yz)$

3. $p$ is prime.
   $(p \in \mathbb{Z}) \land (p > 1) \land \neg \exists x, y (x < p \land y < p \land (xy = p))$
Domains

How does the choice of domain for the following quantified statements affect whether each statement is true or false? The domains to pick from are \( \mathbb{N}, \mathbb{Z}, \mathbb{Q} \) and \( \mathbb{R} \).

1. \( \forall x \exists y (2x - y = 0) \)

2. \( \exists y \forall x (2x - y = 0) \)

3. \( \forall x \exists y (x - 2y = 0) \)
Domains

How does the choice of domain for the following quantified statements affect whether each statement is true or false? The domains to pick from are $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$.

1. $\forall x \exists y (2x - y = 0)$

2. $\exists y \forall x (2x - y = 0)$

3. $\forall x \exists y (x - 2y = 0)$

4. $\forall x (x < 10 \rightarrow \forall y (y < x \rightarrow y < 9))$

5. $\exists x \exists y (x + y = 100)$

6. $\forall x \exists y (y > x \land \exists z (y + z = 100))$
True or False?

1. Domain: all real numbers

\[ P(x, y): x + y = 0 \]

Predicate 1: \( \forall x \exists y P(x, y) \)
Predicate 2: \( \exists x \forall y P(x, y) \)
True or False?

1. Domain: all real numbers
   \[ P(x, y): x + y = 0 \]
   Predicate 1: \( \forall x \exists y P(x, y) \)
   Predicate 2: \( \exists x \forall y P(x, y) \)

2. Domain: all rational numbers
   Predicate: \( \forall x \exists y (x < y \land \neg \exists z (x < z \land z < y)) \)
   What if the domain is all integers?
Quiz: True or False?

• If $P(x) = \text{"x is prime"}$
• $Q(x) = \text{"x is even"}$
• the domain is the natural numbers

1. $P(5) \land Q(10) \land \neg Q(5) \land \neg P(4)$

2. $(\forall x P(x)) \rightarrow Q(4)$

3. $\neg \exists x, y (P(x) \land P(y) \land P(x + y))$

4. $\exists x (P(x) \land Q(x) \land \forall y ((P(y) \land Q(y)) \rightarrow x = y))$

5. $\forall x (\neg P(x) \rightarrow Q(x))$

6. $\forall x ((x > 2 \land P(x)) \rightarrow \exists y (Q(y) \land x = y + 1))$
Assignments for Tuesday

- First homework due at start of class
- Modules 4,5 with associated readings
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