CS311H
Discrete Math for CS: Honors

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Department of Computer Science
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Good Morning, Colleagues

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Are there any questions?

Logistics

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Some important concepts

- Contrapositive vs. contradiction
- How to choose a proof technique
  - How formal is formal enough?
  - Proof as communication. Be able to play *both* roles.
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- For integer n, If n^3 is even, then n is even.

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8. However, according to the Pythagorean Theorem: $c^2 = a^2 + b^2$. Contradicts line 7.

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