Good Morning, Colleagues
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Are there any questions?
Logistics

- Good job on quiz!
Logistics

• Good job on quiz!

• For now, we’ll continue to push fast
Logistics

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- For now, we’ll continue to push fast
- Next week’s assignments up
Logistics

- Good job on quiz!
- For now, we’ll continue to push fast
- Next week’s assignments up
- Send questions to all of us, or use piazza
Logistics

- Good job on quiz!
- For now, we’ll continue to push fast
- Next week’s assignments up
- Send questions to all of us, or use piazza
- Cheating
Some important concepts

• Contrapositive vs. contradiction

• How to choose a proof technique
  • How formal is formal enough?
  • Proof as communication. Be able to play both roles.

• WLOG

• Constructive vs. non-constructive
Some important concepts

• Contrapositive vs. contradiction

• How to choose a proof technique
  • How formal is formal enough?
  • Proof as communication. Be able to play both roles.

• WLOG

• Constructive vs. non-constructive

• Operators and quantifiers we’ll use:
Some important concepts

- Contrapositive vs. contradiction
- How to choose a proof technique
  - How formal is formal enough?
  - Proof as communication. Be able to play both roles.
- WLOG
- Constructive vs. non-constructive
- Operators and quantifiers we’ll use: $\forall, \exists, \neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
Some important concepts

• Contrapositive vs. contradiction

• How to choose a proof technique
  • How formal is formal enough?
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• WLOG

• Constructive vs. non-constructive

• Operators and quantifiers we’ll use: ∀, ∃, ¬, ∧, ∨, ⇒, ⇔

• Domains of numbers:
Some important concepts

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  - How formal is formal enough?
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- Constructive vs. non-constructive
- Operators and quantifiers we’ll use: ∀, ∃, ¬, ∧, ∨, ⇒, ⇔
- Domains of numbers: Z, N, Q, R
Some important concepts

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- How to choose a proof technique
  - How formal is formal enough?
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- WLOG
- Constructive vs. non-constructive
- Operators and quantifiers we’ll use: ∀, ∃, ¬, ∧, ∨, ⇒, ⇔
-Domains of numbers: Z, N, Q, R 0 ∉ N
Prove that...

- The product of an even number and an odd number is always even.
Prove that... 

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• If x and y are two integers whose product is even, then at least one of the two must be even.
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• If $x$ and $y$ are two integers whose product is even, then at least one of the two must be even.
  More formally: For $x, y \in \mathbb{Z}$, $(xy \text{ even}) \implies (x \text{ even } \lor y \text{ even})$
Prove that . . .

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  Use contrapositive: $(x \text{ odd } \land y \text{ odd}) \Rightarrow (xy \text{ odd})$
Prove that...

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- If \( x \) and \( y \) are two integers whose product is even, then at least one of the two must be even.

  More formally: For \( x, y \in \mathbb{Z} \), \((xy \text{ even}) \Rightarrow (x \text{ even} \lor y \text{ even})\)

  Use contrapositive: \((x \text{ odd} \land y \text{ odd}) \Rightarrow (xy \text{ odd})\)

- For integer \( n \), If \( n^3 \) is even, then \( n \) is even.
Prove that...

- Suppose $a$, $b$ and $c$ are integers. If ($b$ is a multiple of $a$) and ($c$ is a multiple of $a$) then ($b + c$ is a multiple of $a$).
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- Suppose $a$, $b$ and $c$ are integers. If ($b$ is a multiple of $a$) and ($c$ is a multiple of $a$) then (($b + c$) is a multiple of $a$).

- The length of the hypotenuse of a right triangle is less than the sum of the lengths of the two legs.
Proof

1. Let $a$, $b$, and $c$ be different lengths of sides of a right triangle, in which $c$ corresponds to the hypotenuse.
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1. Let $a$, $b$, and $c$ be different lengths of sides of a right triangle, in which $c$ corresponds to the hypotenuse.
2. To prove that $c < a + b$, assume the opposite, i.e. $c \geq a + b$, and seek a contradiction.
Proof

1. Let \( a, b, \) and \( c \) be different lengths of sides of a right triangle, in which \( c \) corresponds to the hypotenuse.
2. To prove that \( c < a + b \), assume the opposite, i.e. \( c \geq a + b \), and seek a contradiction.
3. Because \( a, b, \) and \( c \) are triangle side lengths, they are positive numbers.
Proof

1. Let $a$, $b$, and $c$ be different lengths of sides of a right triangle, in which $c$ corresponds to the hypotenuse.
2. To prove that $c < a + b$, assume the opposite, i.e. $c \geq a + b$, and seek a contradiction.
3. Because $a$, $b$, and $c$ are triangle side lengths, they are positive numbers.
4. Therefore, the assumed inequality can be squared to get $c^2 \geq (a + b)^2$. 
Proof

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2. To prove that $c < a + b$, assume the opposite, i.e. $c \geq a + b$, and seek a contradiction.
3. Because $a$, $b$, and $c$ are triangle side lengths, they are positive numbers.
4. Therefore, the assumed inequality can be squared to get $c^2 \geq (a + b)^2$.
5. Expand the right-hand side to get $c^2 \geq a^2 + 2ab + b^2$. 
Proof

1. Let $a$, $b$, and $c$ be different lengths of sides of a right triangle, in which $c$ corresponds to the hypotenuse.
2. To prove that $c < a + b$, assume the opposite, i.e. $c \geq a + b$, and seek a contradiction.
3. Because $a$, $b$, and $c$ are triangle side lengths, they are positive numbers.
4. Therefore, the assumed inequality can be squared to get $c^2 \geq (a + b)^2$.
5. Expand the right-hand side to get $c^2 \geq a^2 + 2ab + b^2$.
6. Because $a$ and $b$ are positive, $a^2 + 2ab + b^2 > a^2 + b^2$. 
Proof

1. Let $a$, $b$, and $c$ be different lengths of sides of a right triangle, in which $c$ corresponds to the hypotenuse.
2. To prove that $c < a + b$, assume the opposite, i.e. $c \geq a + b$, and seek a contradiction.
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4. Therefore, the assumed inequality can be squared to get $c^2 \geq (a + b)^2$.
5. Expand the right-hand side to get $c^2 \geq a^2 + 2ab + b^2$.
6. Because $a$ and $b$ are positive, $a^2 + 2ab + b^2 > a^2 + b^2$.
7. Combining the inequalities in lines 5 and 6 yields $c^2 > a^2 + b^2$. 
Proof

1. Let \( a, b, \) and \( c \) be different lengths of sides of a right triangle, in which \( c \) corresponds to the hypotenuse.
2. To prove that \( c < a + b \), assume the opposite, i.e. \( c \geq a + b \), and seek a contradiction.
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4. Therefore, the assumed inequality can be squared to get \( c^2 \geq (a + b)^2 \).
5. Expand the right-hand side to get \( c^2 \geq a^2 + 2ab + b^2 \).
6. Because \( a \) and \( b \) are positive, \( a^2 + 2ab + b^2 > a^2 + b^2 \).
7. Combining the inequalities in lines 5 and 6 yields \( c^2 > a^2 + b^2 \).
8. However, according to the Pythagorean Theorem: \( c^2 = a^2 + b^2 \).
Proof

1. Let \( a, b, \) and \( c \) be different lengths of sides of a right triangle, in which \( c \) corresponds to the hypotenuse.
2. To prove that \( c < a + b \), assume the opposite, i.e. \( c \geq a + b \), and seek a contradiction.
3. Because \( a, b, \) and \( c \) are triangle side lengths, they are positive numbers.
4. Therefore, the assumed inequality can be squared to get \( c^2 \geq (a + b)^2 \).
5. Expand the right-hand side to get \( c^2 \geq a^2 + 2ab + b^2 \).
6. Because \( a \) and \( b \) are positive, \( a^2 + 2ab + b^2 > a^2 + b^2 \).
7. Combining the inequalities in lines 5 and 6 yields \( c^2 > a^2 + b^2 \).
8. However, according to the Pythagorean Theorem: \( c^2 = a^2 + b^2 \). Contradicts line 7.
Prove that...

• $xy$ is not a multiple of $z \Rightarrow$ neither $x$ nor $y$ is a multiple of $z$. 
Prove that...

- $xy$ is not a multiple of $z \implies$ neither $x$ nor $y$ is a multiple of $z$. More formally:

  $\neg(xy$ mult. of $z) \implies (\neg(x$ mult. of $z) \wedge \neg(y$ mult. of $z))$. 
Prove that...

- $xy$ is not a multiple of $z \Rightarrow$ neither $x$ nor $y$ is a multiple of $z$. More formally:
  $\neg(xy$ mult. of $z) \Rightarrow (\neg(x$ mult. of $z) \land \neg(y$ mult. of $z))$.
  Contrapositive is:
  $((x$ mult. of $z) \lor (y$ mult. of $z)) \rightarrow (xy$ mult. of $z)$. 
Prove that...

- $xy$ is not a multiple of $z$ $\implies$ neither $x$ nor $y$ is a multiple of $z$.
  More formally:
  $\neg(\text{xy mult. of } z) \implies (\neg(\text{x mult. of } z) \wedge \neg(\text{y mult. of } z))$.
  Contrapositive is:
  $((x \text{ mult. of } z) \lor (y \text{ mult. of } z)) \rightarrow (xy \text{ mult. of } z)$. 
Assignments for Thursday

- Look at second homework
- Module 6
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