CS311H

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Odd snowball fight

- An odd number of people stand on a football field. No two people are the same distance from each other as any other two people. When I shout “go”, everyone throws a snowball at his/her nearest neighbor, hitting this person. Prove that at least one person is not hit by a snowball (a “survivor”).

Good Morning, Colleagues

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Are there any questions?

Logistics

- Third homework **due at start of class in a week**

Some questions

- Not all horses have the same color (see Piazza)

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- Base case: n=1 (3 people)

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- Call the two closest people A and B
- There is survivor of other $2k + 1$ (IH)
- That person is still a survivor.

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- For integer $n > 0$: $1^2 + 2^2 + 3^2 + ... + n^2 = (1/6)n(n+1)(2n+1)$

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- For integer $n > 0$: $2^2 + 5^2 + 8^2 + (3n-1)^2 = (1/2)n(6n^2+3n-1)$

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- For integer $n > 0$: $2^2 + 5^2 + 8^2 + (3n-1)^2 = (1/2)n(6n^2 + 3n - 1)$

  Base Case: $2^2 = 4 = 8/2 = (1/2)(6 + 3 - 1)$

  Inductive Step:

  1. $2^2 + 5^2 + 8^2 + (3n-1)^2 + (3(n+1)-1)^2$
  2. $= (1/2)n(6n^2 + 3n - 1) + (3(n+1)-1)^2$ {IH}
  3. $= (1/2)n(6n^2 + 3n - 1) + (2/2)(3n + 3 - 1)^2$
  4. $= (1/2)[n(6n^2 + 3n - 1) + 2(3n+2)^2]$
  5. $= (1/2)[n(6n^2 + 3n - 1) + 2(9n^2 + 12n + 4)]$
  6. $= (1/2)[6n^3 + 3n^2 - n + 18n^2 + 24n + 8]$
  7. $= (1/2)[6n^3 + 21n^2 + 23n + 8]$
  8. $= (1/2)(n+1)(6n^2 + 15n + 8)$
  9. $= (1/2)(n+1)(6n^2 + 15n + 9 - 1)$
  10. $= (1/2)(n+1)(6n^2 + 12n + 6 + 3n + 3 - 1)$

11. $= (1/2)(n+1)(6(n^2+2n+1)+3(n+1)-1)$
12. $= (1/2)(n+1)(6(n+1)^2+3(n+1)-1)$

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  2. $\equiv \neg(X_1 \wedge ... \wedge X_n) \vee \neg X_{n+1}$ {IH}
  3. $\equiv \neg[(X_1 \wedge ... \wedge X_n) \wedge X_{n+1}]$ {De Morgan's Law}

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  3. $\equiv \neg[(X_1 \wedge \ldots \wedge X_n) \wedge X_{n+1}]$ {De Morgan's Law}
  4. $\equiv \neg(X_1 \wedge \ldots \wedge X_n \wedge X_{n+1})$ {Associativity}

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  4. $= n^3 + n^3$

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  5. $\geq n^3 + 10n^2$ {since $n \geq 10$}

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  6. $= n^3 + 3n^2 + 7n^2$

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  7. $\geq n^3 + 3n^2 + 70n$ {since $n \geq 10$}

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  8. $= n^3 + 3n^2 + 3n + 67n$

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  9. $> n^3 + 3n^2 + 3n + 1$

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  9. $> n^3 + 3n^2 + 3n + 1$
  10. $= (n+1)^3$

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  5. $4^{n+1} + 24n - 4 - 18n + 9$

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  6. $4(4^n + 6n - 1) + 9(1 - 2n)$

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  5. $4^{n+1} + 24n - 4 - 18n + 9$
  6. $4(4^n + 6n - 1) + 9(1 - 2n)$
  7. $4(9k) + 9(1 - 2n)$ {IH: k is an integer}
  8. $9(4k + 1 - 2n)$

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  4. $4^{n+1} + 6n + 18n - 18n + 5 - 4 + 4$
  5. $4^{n+1} + 24n - 4 - 18n + 9$
  6. $4(4^n + 6n - 1) + 9(1 - 2n)$
  7. $4(9k) + 9(1 - 2n)$ {IH: k is an integer}
  8. $9(4k + 1 - 2n)$
  9. $(4k + 1 - 2n)$ is an integer, so quantity is divisible by 9.

Prove:

- For integer $n > 0$: $\frac{1}{1^2} + \frac{1}{2^2} + ... + \frac{1}{n^2} \leq 2 - \frac{1}{n}$,
  given the fact:
  $[k \geq 1] \rightarrow [\frac{1}{k(k+1)} \geq \frac{1}{(k+1)^2}]$

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  3. $\frac{1}{1^2} + \frac{1}{2^2} + ... + \frac{1}{k^2} + \frac{1}{(k+1)^2} \leq 2 - \frac{k+1}{k(k+1)} + \frac{1}{k(k+1)}$

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  4. $\frac{1}{1^2} + \frac{1}{2^2} + ... + \frac{1}{k^2} + \frac{1}{(k+1)^2} \leq 2 - \frac{k}{k(k+1)}$ {Add}

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  4. $\frac{1}{1^2} + \frac{1}{2^2} + ... + \frac{1}{k^2} + \frac{1}{(k+1)^2} \leq 2 - \frac{k}{k(k+1)}$ {Add}
  5. $\frac{1}{1^2} + \frac{1}{2^2} + ... + \frac{1}{k^2} + \frac{1}{(k+1)^2} \leq 2 - \frac{1}{k+1}$ {Divide}

Prove:

- $(n^2 - 1)$ is divisible by 8 whenever n is an odd positive integer

Assignments for Thursday

- Modules 9 on strong induction

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- Modules 9 on strong induction
- Work on third homework **due at start of class** next Tuesday