CS311H

Prof: Peter Stone

Department of Computer Science
The University of Texas at Austin

Tiling a holey checkerboard

- Let n be a positive integer. Show that every $2^n \times 2^n$ checkerboard with one square removed can be tiled using right triominoes (pieces shaped like the letter "L") that cover three squares of the board.

Good Morning, Colleagues

Good Morning, Colleagues

Are there any questions?

Logistics

- Keeping up and posting on piazza is **required**
  - (If you didn't believe me last week, I hope you do now!)

Logistics

- Keeping up and posting on piazza is **required**
  - (If you didn't believe me last week, I hope you do now!)
  - In general, you should be able to **do** (not just follow) any proofs covered in the modules, class, discussion, or piazza.

Logistics

- Keeping up and posting on piazza is **required**
  - (If you didn't believe me last week, I hope you do now!)
  - In general, you should be able to **do** (not just follow) any proofs covered in the modules, class, discussion, or piazza.
  - Some of them are too hard for us to ask on a quiz or test without prior exposure – don't worry if you couldn't come up with them on your own. But **none** are too hard to ask after you've seen them.

Logistics

- Keeping up and posting on piazza is **required**
  - (If you didn't believe me last week, I hope you do now!)
  - In general, you should be able to **do** (not just follow) any proofs covered in the modules, class, discussion, or piazza.
  - Some of them are too hard for us to ask on a quiz or test without prior exposure – don't worry if you couldn't come up with them on your own. But **none** are too hard to ask after you've seen them.
  - (There will also be new problems on quizzes and tests, but they'll generally be easier).

Logistics

- Keeping up and posting on piazza is **required**
  - (If you didn't believe me last week, I hope you do now!)
  - In general, you should be able to **do** (not just follow) any proofs covered in the modules, class, discussion, or piazza.
  - Some of them are too hard for us to ask on a quiz or test without prior exposure – don't worry if you couldn't come up with them on your own. But **none** are too hard to ask after you've seen them.
  - (There will also be new problems on quizzes and tests, but they'll generally be easier).
- Third homework **due at start of class**

Some questions

- Difference between weak and strong induction

Tricky problem from Piazza

- Prove that when $n \geq 1, a_i \in \mathbb{R}, a_i > 0$, if $a_1 \times a_2 \times ... \times a_n = 1$, then

$$(1 + a_1)(1 + a_2)...(1 + a_n) \geq 2^n$$

Strong induction

- (5.2.3) Any amount of postage more than 7 cents can be paid for with a combination of 3 and 5 cent stamps.

Strong induction

- (5.2.3) Any amount of postage more than 7 cents can be paid for with a combination of 3 and 5 cent stamps. $P(n) = 3k + 5q$ for integer k and q.

Strong induction

- (5.2.3) Any amount of postage more than 7 cents can be paid for with a combination of 3 and 5 cent stamps.
  $P(n) = 3k + 5q$ for integer k and q.
  Base Case 0: $8 = 5 + 3$
  Base Case 1: $9 = 3 + 3 + 3$
  Base Case 2: $10 = 5 + 5$

Strong induction

- (5.2.3) Any amount of postage more than 7 cents can be paid for with a combination of 3 and 5 cent stamps.
  $P(n) = 3k + 5q$ for integer k and q.
  Base Case 0: $8 = 5 + 3$
  Base Case 1: $9 = 3 + 3 + 3$
  Base Case 2: $10 = 5 + 5$
  Inductive Case: Allowed to assume $n \geq 10$, and $P(k)$ for $8 \leq k \leq n$

Strong induction

- (5.2.3) Any amount of postage more than 7 cents can be paid for with a combination of 3 and 5 cent stamps.
  $P(n) = 3k + 5q$ for integer k and q.
  Base Case 0: $8 = 5 + 3$
  Base Case 1: $9 = 3 + 3 + 3$
  Base Case 2: $10 = 5 + 5$
  Inductive Case: Allowed to assume $n \geq 10$, and $P(k)$ for $8 \leq k \leq n$
  1. $n + 1 = n - 2 + 3$

Strong induction

- (5.2.3) Any amount of postage more than 7 cents can be paid for with a combination of 3 and 5 cent stamps.
  $P(n) = 3k + 5q$ for integer k and q.
  Base Case 0: $8 = 5 + 3$
  Base Case 1: $9 = 3 + 3 + 3$
  Base Case 2: $10 = 5 + 5$
  Inductive Case: Allowed to assume $n \geq 10$, and $P(k)$ for $8 \leq k \leq n$
  1. $n + 1 = n - 2 + 3$
  2. $n \geq 10 \Rightarrow n - 2 \geq 8$

Strong induction

- (5.2.3) Any amount of postage more than 7 cents can be paid for with a combination of 3 and 5 cent stamps.
  $P(n) = 3k + 5q$ for integer k and q.
  Base Case 0: $8 = 5 + 3$
  Base Case 1: $9 = 3 + 3 + 3$
  Base Case 2: $10 = 5 + 5$
  Inductive Case: Allowed to assume $n \geq 10$, and $P(k)$ for $8 \leq k \leq n$
  1. $n + 1 = n - 2 + 3$
  2. $n \geq 10 \Rightarrow n - 2 \geq 8$
  3. Since $n \geq n - 2 \geq 8$, $P(n-2)$ is true

Strong induction

- (5.2.3) Any amount of postage more than 7 cents can be paid for with a combination of 3 and 5 cent stamps.
  $P(n) = 3k + 5q$ for integer k and q.
  Base Case 0: $8 = 5 + 3$
  Base Case 1: $9 = 3 + 3 + 3$
  Base Case 2: $10 = 5 + 5$
  Inductive Case: Allowed to assume $n \geq 10$, and $P(k)$ for $8 \leq k \leq n$
  1. $n + 1 = n - 2 + 3$
  2. $n \geq 10 \Rightarrow n - 2 \geq 8$
  3. Since $n \geq n - 2 \geq 8$, $P(n - 2)$ is true
  4. So, $n + 1 = (n - 2) + 3 = 3i + 5j + 3 = 3(i + 1) + 5j$

Strong induction

- (5.2.3) Any amount of postage more than 7 cents can be paid for with a combination of 3 and 5 cent stamps.
  $P(n) = 3k + 5q$ for integer k and q.
  Base Case 0: $8 = 5 + 3$
  Base Case 1: $9 = 3 + 3 + 3$
  Base Case 2: $10 = 5 + 5$
  Inductive Case: Allowed to assume $n \geq 10$, and $P(k)$ for $8 \leq k \leq n$
  1. $n + 1 = n - 2 + 3$
  2. $n \geq 10 \Rightarrow n - 2 \geq 8$
  3. Since $n \geq n - 2 \geq 8$, $P(n - 2)$ is true
  4. So, $n + 1 = (n - 2) + 3 = 3i + 5j + 3 = 3(i + 1) + 5j$
  5. So, $n + 1$ cents can be paid with 3 and 5 cent stamps.

Tiling a holey checkerboard

- Let n be a positive integer. Show that every $2^n \times 2^n$ checkerboard with one square removed can be tiled using right triominoes (pieces shaped like the letter "L") that cover three squares of the board.

Tiling a holey checkerboard

- Let n be a positive integer. Show that every $2^n \times 2^n$ checkerboard with one square removed can be tiled using right triominoes (pieces shaped like the letter "L") that cover three squares of the board.

- (see proof in the book)

Strong induction

- Which amounts of money can be formed using just \$2 and \$5 bills?

More tiling

- Prove: When $n > 1$, Assume we have three kinds of tiles: 1 by 2 tiles, 2 by 1 tiles and 2 by 2 tiles. Prove given a n by 2 board, there are

$$\frac{2^{n+1} + (-1)^n}{3}$$

ways to fill it using these three kinds of tiles.

Prove:

- Every positive integer can be written as a sum of distinct powers of 2

Assignments for Tuesday

- Third homework **due at start of class**
- Modules 15,16 with associated readings

Assignments for Tuesday

- Third homework **due at start of class**

- Modules 15,16 with associated readings