CS311H

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Are there any questions?

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  - Handwritten notes allowed
  - No book, nothing printed, nothing electronic
  - Be on time!

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- HW4 review

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$$f(x) = \begin{cases} 1/2 & \text{if } x = 0 & \text{(case 1)} \\ x/(1+x) & \text{if } \exists n \in \mathbb{N}[x = 1/n] & \text{(case 2)} \\ x & \text{otherwise} & \text{(case 3)} \end{cases}$$

Propositions

- Satisfiable? $\neg(\neg X_1 \vee X_2) \wedge \neg(X_2 \vee X_3 \vee (\neg X_3 \wedge \neg(X_1 \vee X_2)))$ If so, produce a satisfying assignment. If not, prove it is unsatisfiable.

- Prove $\neg X_1 \vee (X_1 \wedge \neg(X_2 \rightarrow \neg X_1)) \equiv \neg X_1 \vee X_2$.

Translation

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   $P(x)$: x has blue eyes
   $Q(x)$: x has black eyes
   Statement: There exist people with blue eyes and with black eyes, but one cannot have blue and black eyes at the same time.

2. Domain: all UT student
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2. Domain: all UT student
   $P(x)$: x is a computer science student
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   Statement: $\forall x (P(x) \rightarrow Q(x))$
   Answer: All computer science students in UT must take 311

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- Sponge: Suppose $A \subseteq B \subseteq C$. Prove $C - B \subseteq C - A$.

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Injection, surjection, both or neither?

- Let $f : R \to R$ be function that $f(x) = x^2 - 2x - 2$
- Sponge: $f : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}, f(a, b) = 2^a 3^b$

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- If r is any rational number and if s is any irrational number, then r/s is irrational.
  Let $r = 0$, $s = \pi$, then $r/s = 0$, which is rational.

Assignments for Thursday

- Modules 10 and 11