CS313H

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Warm ups

• How many sequences of 7 digits have at least one repeating digit?

• How many ways are there to arrange the letters in the word “SYSTEMS”?

• How many hands of 5 cards have at least 3 aces?
Good Morning, Colleagues
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Are there any questions?
Logistics

- Class survey
Logistics

- Class survey
  - Don’t like 8pm quest deadline
Logistics

- Class survey
  - Don’t like 8pm quest deadline
  - Flipped class
Logistics

- Class survey
  - Don’t like 8pm quest deadline
  - Flipped class

- Modules C2 and 17 for Tuesday
Logistics

- Class survey
  - Don’t like 8pm quest deadline
  - Flipped class

- Modules C2 and 17 for Tuesday

- Homework 6 due Thursday (+ Module 18)
Logistics

- Class survey
  - Don’t like 8pm quest deadline
  - Flipped class

- Modules C2 and 17 for Tuesday

- Homework 6 due Thursday (+ Module 18)
  - Homework 7 due following Tuesday
Logistics

• Class survey
  – Don’t like 8pm quest deadline
  – Flipped class

• Modules C2 and 17 for Tuesday

• Homework 6 due Thursday (+ Module 18)
  – Homework 7 due following Tuesday

• Midterm on graph theory, counting, recurrences following Thursday
Important counting concepts

- Addition rule
- Inclusion/exclusion principle
- Correspondence principle
- Product rule

- Number of subsets of an \( n \) element set: \( 2^n \)
- Number of permutations of \( n \) distinct objects: \( n! \)
- Number of subsets of size \( k \) from an \( n \)-element set: \( \binom{n}{k} \) ("n choose k") = \( \frac{n!}{k!(n-k)!} \)
Counting trees and graphs

- How many trees with 4 nodes?
  - unlabeled vs. labeled
Counting trees and graphs

- How many trees with 4 nodes?
  - unlabeled vs. labeled
  - 2, 16
Counting trees and graphs

- How many trees with 4 nodes?
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- How many labeled graphs on \( n \) nodes?
Counting trees and graphs

• How many trees with 4 nodes?
  – unlabeled vs. labeled
  – 2, 16

• How many labeled graphs on $n$ nodes?
  – $2^{\binom{n}{2}}$
Counting trees and graphs

- How many trees with 4 nodes?
  - unlabeled vs. labeled
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- How many labeled graphs on $n$ nodes?
  - $2^{n \choose 2}$

- How many labeled trees on $n$ nodes?
Counting trees and graphs

- How many trees with 4 nodes?
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- How many labeled graphs on $n$ nodes?
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- How many labeled trees on $n$ nodes?
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Warm ups

• How many sequences of 7 digits have at least one repeating digit?

• How many ways are there to arrange the letters in the word “SYSTEMS”?

• How many hands of 5 cards have at least 3 aces?
Counting Poker Hands

• Each card has 13 possible ranks
Counting Poker Hands

- Each card has 13 possible ranks
- AND 4 possible suits
Counting Poker Hands

- Each card has 13 possible ranks
- AND 4 possible suits
- A straight is a sequence of 5 cards of consecutive rank
Counting Poker Hands

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- a **straight** is a sequence of 5 cards of consecutive rank
- a **flush** is a set of 5 cards with the same suit
Counting Poker Hands

- Each card has 13 possible ranks
- AND 4 possible suits

- a **straight** is a sequence of 5 cards of consecutive rank
- a **flush** is a set of 5 cards with the same suit

- How many hands total?
Counting Poker Hands

- Each card has 13 possible ranks
- AND 4 possible suits

- a **straight** is a sequence of 5 cards of consecutive rank
- a **flush** is a set of 5 cards with the same suit

- How many hands total? \( \binom{52}{5} = 2,598,960 \)
How many? What are the odds?

**Straight flush:** straight and a flush

**4 of a kind:** 4 cards of the same rank

**Full house:** 3 cards of one rank, two of another

**Flush:** a flush but *not* a straight

**Straight:** a straight but *not* a flush

**3 of a kind:** 3 cards of one rank, but not full house or 4 of a kind

**2 pair:** 2 cards of one rank, 2 of another rank, but *not* 4 of a kind or full house

**Pair:** 2 cards of one rank, but not anything higher
How many? What are the odds?

Straight flush:
How many? What are the odds?

**Straight flush:** \[
\frac{36}{2,598,960} = .0000138 \approx 1 \text{ in } 72,193
\]
How many? What are the odds?

**Straight flush:** \( \frac{36}{2,598,960} = .0000138 \approx 1 \text{ in } 72,193 \)

4 of a kind:
How many? What are the odds?

**Straight flush:** \( \frac{36}{2,598,960} = .0000138 \approx 1 \text{ in } 72,193 \)

**4 of a kind:** \( \frac{624}{2,598,960} = .00024 = 1 \text{ in } 4165 \)
How many? What are the odds?

Straight flush: \[ \frac{36}{2,598,960} = 0.0000138 \approx 1 \text{ in } 72,193 \]

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full house:
How many? What are the odds?

**Straight flush:** \( \frac{36}{2,598,960} = .0000138 \approx 1 \text{ in } 72,193 \)

**4 of a kind:** \( \frac{624}{2,598,960} = .00024 \approx 1 \text{ in } 4165 \)

**Full house:** \( \frac{3744}{2,598,960} = .00144 \approx 1 \text{ in } 694 \)
How many? What are the odds?

Straight flush: \( \frac{36}{2,598,960} = 0.000138 \approx \frac{1}{72,193} \)

4 of a kind: \( \frac{624}{2,598,960} = 0.00024 = \frac{1}{4165} \)

full house: \( \frac{3744}{2,598,960} = 0.00144 \approx \frac{1}{694} \)

flush:
How many? What are the odds?

**Straight flush:** \( \frac{36}{2,598,960} = 0.0000138 \approx 1 \text{ in 72,193} \)

**4 of a kind:** \( \frac{624}{2,598,960} = 0.00024 = 1 \text{ in 4165} \)

**Full house:** \( \frac{3744}{2,598,960} = 0.00144 \approx 1 \text{ in 694} \)

**Flush:** \( \frac{5112}{2,598,960} = 0.0019669 \approx 1 \text{ in 508} \)
How many? What are the odds?

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**Flush:** \( \frac{5112}{2,598,960} = .0019669 \approx 1 \text{ in } 508 \)

**Straight:** \( \frac{9180}{2,598,960} = .00353 \approx 1 \text{ in } 283 \)
How many? What are the odds?

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**3 of a kind:**
How many? What are the odds?

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**3 of a kind:** \[
\frac{54,912}{2,598,960} = .0211 \approx 1 \text{ in } 47
\]
How many? What are the odds?

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2 pair:
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**3 of a kind:** \( \frac{54,912}{2,598,960} = .0211 \approx 1 \text{ in } 47 \)

**2 pair:** \( \frac{123,552}{2,598,960} = .0475 \approx 1 \text{ in } 21 \)
How many? What are the odds?

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**Pair:** \( \frac{1,098,240}{2,598,960} = .4225 \approx 1 \text{ in } 2.4 \)