CS311H

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Warm Up

- **REVIEW:** How many sequences (permutations) are there of 5 different bills from among 100,50,20,10,5,2,1?
- **REVIEW:** How many ways are there to select 5 bills from 100,50,20,10,5,2,1? (You can only select each type once)
- **NEW:** How many ways are there to select 5 bills from 100,50,20,10,5,2,1? (You can select any type as many times as you want)

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Are there any questions?

Logistics

- Did you work through Eulerian graph proofs?
- Have you figured out the poker hands?

Some questions

- A few more facts about Pascal's triangle

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- How many non negative integer solutions are there to $p + q + r < 11$? $\binom{13}{3} = 286$

Pigeonhole Principle

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- (HARD) In a month with 30 days, a baseball team will play 45 games. It must also play at least one game on each day. Show that there will be a period of *consecutive* days where exactly 14 games are played.

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- Coefficient of $x_1^{r_1}x_2^{r_2}\cdots x_k^{r_k}$ in $(x_1+x_2+\cdots x_k)^n$ = $\frac{n!}{r_1!r_2!\cdots r_n!}$