CS313H
Logic, Sets, and Functions: Honors
Fall 2012

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Good Morning, Colleagues

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Are there any questions?

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- Vote!

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- Wed. before Thanksgiving?

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  – v can be give a new color, which means X(G) <= k-1, contradiction.

Counting

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- How many ways to choose a dozen donuts if there are 4 types?

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- How many ways are there to place 35 students into 7 groups of 5?

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  - $3^6 - (\binom{3}{1}2^6 - \binom{3}{2})$ = 540

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- Left equation: pick a leader first from n, then there are 2^{n-1} possible subsets of other people.
- Right equation: consider how many committees of size k there are from $k = 1$ to n. For each of these, there are k possible leaders.

Soccer Balls

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- Suppose that a planar graph with E edges and V vertices contains no simple circuits of length 4 or less.
Show that E $\leq \frac{5}{3}V - \frac{10}{3}$ if $V \geq 4$

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Thus we have $x = 12$. So we have 12 pentagons.