CS313H
Logic, Sets, and Functions: Honors
Fall 2012

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Are there any questions?

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- Class Tuesday next week important
- No discussion Wed. before Thanksgiving

Important Points

- How does O,Ω,Θ relate to limits?
- $f(x)$ being of "order" $g(x)$ is a way of saying $f(x)$ is $\Theta(g(x))$

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  4. $< (x^2+x^2)/x$ {because $x > 1$}
  5. $= 2x^2/x$
  6. $= 2x$
  7. $= 2|x|$
  8. Therefore $C = 2$ and $\forall x > K$, $|(x^2+1)/(x+1)| \leq C|x|$.

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  But $n^3 \leq C7n^2$ implies that $n \leq 7C$
  But this fails for values of n that are greater than 7C. So we have a contradiction.

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  - (Try on piazza)

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- Show that $f(n)$ is neither $O(1)$ **nor** $\Omega(1)$
- Find a function $g(n)$ such that $f(n)$ is $\Theta(g(n))$.

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  - $g(n) = n(\sin n)$
  - Every function is Θ of itself!