Good Morning, Colleagues
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Are there any questions?
Logistics

- Midterm was difficult and a bit too long
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  - Acts as a separator — will aim for final to be roughly same difficulty, but not as time-pressured
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  - Don’t despair about grades
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- Until Thanksgiving: Big O and Master Theorem
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• Until Thanksgiving: Big O and Master Theorem
  – This week may have been review – consider it vacation after exam
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• Class Tuesday next week important
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- Class Tuesday next week important

- No discussion Wed. before Thanksgiving
Important Points

- How does $O, \Omega, \Theta$ relate to limits?
- $f(x)$ being of “order” $g(x)$ is a way of saying $f(x)$ is $\Theta(g(x))$
Prove (and find C and K)

- $f(n) = 4n^2 - 5n + 3$ is $O(n^2)$.
Prove (and find C and K)

- $f(n) = 4n^2 - 5n + 3$ is $O(n^2)$.
- $f(n) = (n + 5)\log_2(3n^2 + 7)$ is $O(n \log_2 n)$. 
Prove (and find C and K)

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• $f(n) = (n + 5)\log_2(3n^2 + 7)$ is $O(n \log_2 n)$.

• $(x^2 + 1)/(x + 1)$ is $O(x)$.
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- $(x^2 + 1)/(x + 1)$ is $O(x)$.
  1. Let $K = 1$. 

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Prove (and find C and K)

- $f(n) = 4n^2 - 5n + 3$ is $O(n^2)$.
- $f(n) = (n + 5)\log_2(3n^2 + 7)$ is $O(n \log_2 n)$.
- $(x^2 + 1)/(x + 1)$ is $O(x)$.
  1. Let $K = 1$.
  2. For $x > K$, $|(x^2 + 1)/(x + 1)| = (x^2 + 1)/(x + 1)$. 

Peter Stone
Prove (and find C and K)

- \( f(n) = 4n^2 - 5n + 3 \) is \( O(n^2) \).

- \( f(n) = (n + 5)\log_2(3n^2 + 7) \) is \( O(n \log_2 n) \).

- \( \frac{x^2 + 1}{x + 1} \) is \( O(x) \).
  1. Let \( K = 1 \).
  2. For \( x > K \), \( |\frac{x^2 + 1}{x + 1}| = \frac{x^2 + 1}{x + 1} \).
  3. \( \frac{x^2 + 1}{x + 1} < \frac{x^2 + 1}{x} \)
Prove (and find C and K)

- $f(n) = 4n^2 - 5n + 3$ is $O(n^2)$.

- $f(n) = (n + 5)\log_2(3n^2 + 7)$ is $O(n \log_2 n)$.

- $\frac{x^2 + 1}{x + 1}$ is $O(x)$.
  1. Let $K = 1$.
  2. For $x > K$, $|\frac{x^2 + 1}{x + 1}| = \frac{x^2 + 1}{x + 1}$.
  3. $\frac{x^2 + 1}{x + 1} < \frac{x^2 + 1}{x}$
  4. $< \frac{x^2 + x^2}{x}$
Prove (and find C and K)

- \( f(n) = 4n^2 - 5n + 3 \) is \( O(n^2) \).

- \( f(n) = (n + 5)\log_2(3n^2 + 7) \) is \( O(n \log_2 n) \).

- \( \frac{x^2 + 1}{x + 1} \) is \( O(x) \).
  1. Let \( K = 1 \).
  2. For \( x > K \), \( |(x^2 + 1)/(x + 1)| = (x^2 + 1)/(x + 1) \).
  3. \( (x^2 + 1)/(x + 1) < (x^2 + 1)/x \)
  4. \( < (x^2 + x^2)/x \) \{because \( x > 1 \}\)
Prove (and find C and K)

- \( f(n) = 4n^2 - 5n + 3 \) is \( O(n^2) \).

- \( f(n) = (n + 5)\log_2(3n^2 + 7) \) is \( O(n \log_2 n) \).

- \( \frac{x^2 + 1}{x + 1} \) is \( O(x) \).
  1. Let \( K = 1 \).
  2. For \( x > K \), \( |\frac{x^2 + 1}{x + 1}| = \frac{x^2 + 1}{x + 1} \).
  3. \( \frac{x^2 + 1}{x + 1} < \frac{x^2 + 1}{x} \)
  4. \( \quad < \frac{x^2 + x^2}{x} \) \quad \{\text{because } x > 1\}
  5. \( \quad = \frac{2x^2}{x} \)
Prove (and find C and K)

• \( f(n) = 4n^2 - 5n + 3 \) is \( O(n^2) \).

• \( f(n) = (n + 5)\log_2(3n^2 + 7) \) is \( O(n \log_2 n) \).

• \( \frac{x^2 + 1}{x + 1} \) is \( O(x) \).
  1. Let \( K = 1 \).
  2. For \( x > K \), \( |\frac{x^2 + 1}{x + 1}| = \frac{x^2 + 1}{x + 1} \).
  3. \( \frac{x^2 + 1}{x + 1} < \frac{x^2 + 1}{x} \)
  4. \( < \frac{x^2 + x^2}{x} \) \{because \( x > 1 \}\)
  5. \( = 2x^2/x \)
  6. \( = 2x \)
Prove (and find C and K)

- \( f(n) = 4n^2 - 5n + 3 \) is \( O(n^2) \).

- \( f(n) = (n + 5)\log_2(3n^2 + 7) \) is \( O(n \log_2 n) \).

- \( \frac{x^2 + 1}{x + 1} \) is \( O(x) \).
  1. Let \( K = 1 \).
  2. For \( x > K \), \( |\frac{x^2 + 1}{x + 1}| = \frac{x^2 + 1}{x + 1} \).
  3. \( \frac{x^2 + 1}{x + 1} < \frac{x^2 + 1}{x} \)
  4. \( < \frac{x^2 + x^2}{x} \) \{because \( x > 1 \}\}
  5. \( = \frac{2x^2}{x} \)
  6. \( = 2x \)
  7. \( = 2|x| \)
Prove (and find C and K)

• \( f(n) = 4n^2 - 5n + 3 \) is \( O(n^2) \).

• \( f(n) = (n + 5)\log_2(3n^2 + 7) \) is \( O(n \log_2 n) \).

• \( \frac{x^2 + 1}{x + 1} \) is \( O(x) \).
  1. Let \( K = 1 \).
  2. For \( x > K \), \(|(x^2 + 1)/(x + 1)| = (x^2 + 1)/(x + 1)\).
  3. \( (x^2 + 1)/(x + 1) < (x^2 + 1)/x \)
  4. \( \quad < (x^2 + x^2)/x \quad \{\text{because } x > 1\} \)
  5. \( \quad = 2x^2/x \)
  6. \( \quad = 2x \)
  7. \( \quad = 2|x| \)
  8. Therefore \( C = 2 \) and \( \forall x > K, |(x^2 + 1)/(x + 1)| \leq C|x| \).
Not big-O

• Show that $n^3$ is not $O(7n^2)$
Not big-O

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Proof by Contradiction:
• Show that $n^3$ is not $O(7n^2)$

Proof by Contradiction:
Suppose $n^3$ is $O(7n^2)$
Show that \( n^3 \) is not \( O(7n^2) \)

Proof by Contradiction:
Suppose \( n^3 \) is \( O(7n^2) \)
Then there are \( C \) and \( k \) such that
\[
    n^3 \leq C'7n^2, \quad \forall n \geq k
\]
Show that $n^3$ is not $O(7n^2)$

Proof by Contradiction:
Suppose $n^3$ is $O(7n^2)$
Then there are $C$ and $k$ such that
\[ n^3 \leq C7n^2, \quad \forall n \geq k \]
But $n^3 \leq C7n^2$ implies that $n \leq 7C$
Show that \( n^3 \) is not \( O(7n^2) \)

Proof by Contradiction:
Suppose \( n^3 \) is \( O(7n^2) \)
Then there are \( C \) and \( k \) such that
\[
n^3 \leq C \cdot 7n^2, \quad \forall n \geq k
\]
But \( n^3 \leq C \cdot 7n^2 \) implies that \( n \leq 7C \)
But this fails for values of \( n \) that are greater than \( 7C \). So we have a contradiction.
General Facts

- Suppose $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$. Prove $f(x)$ is $O(h(x))$. 

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General Facts

• Suppose $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$. Prove $f(x)$ is $O(h(x))$.

• Prove if $f(x)$ is $O(g(x))$, then $g(x)$ is $\Omega(f(x))$
General Facts

- Suppose $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$. Prove $f(x)$ is $O(h(x))$. 

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General Facts

- Suppose $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$. Prove $f(x)$ is $O(h(x))$.

  1. $f(x)$ is $O(g(x)) \Rightarrow \forall x > K_1 |f(x)| \leq C_1 |g(x)|$ for some $K_1, C_1$. 

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General Facts

• Suppose \( f(x) \) is \( O(g(x)) \) and \( g(x) \) is \( O(h(x)) \). Prove \( f(x) \) is \( O(h(x)) \).

1. \( f(x) \) is \( O(g(x)) \) \( \Rightarrow \) \( \forall x > K_1 \| f(x) \| \leq C_1 \| g(x) \| \) for some \( K_1, C_1 \).

2. \( g(x) \) is \( O(h(x)) \) \( \Rightarrow \) \( \forall x > K_2 \| g(x) \| \leq C_2 \| h(x) \| \) for some \( K_2, C_2 \).
Suppose $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$. Prove $f(x)$ is $O(h(x))$.

1. $f(x)$ is $O(g(x)) \Rightarrow \forall x > K_1 |f(x)| \leq C_1 |g(x)|$ for some $K_1, C_1$.
2. $g(x)$ is $O(h(x)) \Rightarrow \forall x > K_2 |g(x)| \leq C_2 |h(x)|$ for some $K_2, C_2$.
3. Let $K = \max(K_1, K_2)$ and $C = C_1 C_2$. 
Suppose $f(x)$ is $O(g(x))$ and $g(x)$ is $O(h(x))$. Prove $f(x)$ is $O(h(x))$.

1. $f(x)$ is $O(g(x))$ $\Rightarrow \forall x > K_1 \left| f(x) \right| \leq C_1 \left| g(x) \right|$ for some $K_1, C_1$.
2. $g(x)$ is $O(h(x))$ $\Rightarrow \forall x > K_2 \left| g(x) \right| \leq C_2 \left| h(x) \right|$ for some $K_2, C_2$.
3. Let $K = \max(K_1, K_2)$ and $C = C_1 C_2$.
4. Then $\forall x > K \left| f(x) \right| \leq C_1 \left| g(x) \right| \leq C_1 (C_2 \left| h(x) \right|) = C \left| h(x) \right|$. 

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Suppose \( f(x) \) is \( O(g(x)) \) and \( g(x) \) is \( O(h(x)) \). Prove \( f(x) \) is \( O(h(x)) \).

1. \( f(x) \) is \( O(g(x)) \) \( \Rightarrow \) \( \forall x > K_1 |f(x)| \leq C_1 |g(x)| \) for some \( K_1, C_1 \).
2. \( g(x) \) is \( O(h(x)) \) \( \Rightarrow \) \( \forall x > K_2 |g(x)| \leq C_2 |h(x)| \) for some \( K_2, C_2 \).
3. Let \( K = \max(K_1, K_2) \) and \( C = C_1 C_2 \).
4. Then \( \forall x > K |f(x)| \leq C_1 |g(x)| \leq C_1 (C_2 |h(x)|) = C |h(x)| \).
5. Therefore \( f(x) \) is \( O(h(x)) \).
General Facts

• Suppose \( f(x) \) is \( O(g(x)) \) and \( g(x) \) is \( O(h(x)) \). Prove \( f(x) \) is \( O(h(x)) \).
  1. \( f(x) \) is \( O(g(x)) \) \( \Rightarrow \) \( \forall x > K_1 |f(x)| \leq C_1 |g(x)| \) for some \( K_1, C_1 \).
  2. \( g(x) \) is \( O(h(x)) \) \( \Rightarrow \) \( \forall x > K_2 |g(x)| \leq C_2 |h(x)| \) for some \( K_2, C_2 \).
  3. Let \( K = \max(K_1, K_2) \) and \( C = C_1 C_2 \).
  4. Then \( \forall x > K |f(x)| \leq C_1 |g(x)| \leq C_1 (C_2 |h(x)|) = C |h(x)| \).
  5. Therefore \( f(x) \) is \( O(h(x)) \).

• Prove if \( f(x) \) is \( O(g(x)) \), then \( g(x) \) is \( \Omega(f(x)) \)
  – (Try on piazza)
Oscillating Functions

- Consider $f(n) = n \sin n$
Oscillating Functions

• Consider $f(n) = n \sin n$

• Show that $f(n)$ is $O(n)$. 

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Oscillating Functions

- Consider $f(n) = n \sin n$

- Show that $f(n)$ is $O(n)$.

- Is $f(n) \Omega(n)$?
Oscillating Functions

• Consider $f(n) = n\sin n$

• Show that $f(n)$ is $O(n)$.

• Is $f(n) \Omega(n)$?

• Show that $f(n)$ is $\Omega(\sin n)$.
Oscillating Functions

• Consider \( f(n) = n \sin(n) \)

• Show that \( f(n) \) is \( O(n) \).

• Is \( f(n) \) \( \Omega(n) \)?

• Show that \( f(n) \) is \( \Omega(\sin(n)) \).

• Is \( f(n) \) \( O(\sin(n)) \)?
Oscillating Functions

- Consider $f(n) = n(\sin n)$

- Show that $f(n)$ is $O(n)$.

- Is $f(n)$ $\Omega(n)$?

- Show that $f(n)$ is $\Omega(\sin n)$.

- Is $f(n)$ $O(\sin n)$?

- Show that $f(n)$ is neither $O(1)$ nor $\Omega(1)$

- Find a function $g(n)$ such that $f(n)$ is $\Theta(g(n))$. 
Oscillating Functions

- Consider \( f(n) = n\sin(n) \)
- Show that \( f(n) \) is \( O(n) \).
- Is \( f(n) \) \( \Omega(n) \)?
- Show that \( f(n) \) is \( \Omega(\sin(n)) \).
- Is \( f(n) \) \( O(\sin(n)) \)?
- Show that \( f(n) \) is neither \( O(1) \) nor \( \Omega(1) \).
- Find a function \( g(n) \) such that \( f(n) \) is \( \Theta(g(n)) \).
  - \( g(n) = n\sin(n) \)
  - Every function is \( \Theta \) of itself!