CS313H
Logic, Sets, and Functions: Honors
Fall 2012

Prof: Peter Stone

TA: Jacob Schrum

Proctor: Sudheesh Katkam

Department of Computer Science
The University of Texas at Austin

Good Morning, Colleagues

Good Morning, Colleagues

Are there any questions?

Logistics

- Office hours delayed

Logistics

- Office hours delayed
- Grades

Logistics

- Office hours delayed
- Grades
- Extra big-O problems on piazza

Logistics

- Office hours delayed
- Grades
- Extra big-O problems on piazza
- Class Tuesday next week important
- No discussion Wed. before Thanksgiving

Questions

- log:

Questions

- log:remember that $\log_b x = \frac{\log_d x}{\log_d b}$

Questions

- log:remember that $\log_b x = \frac{\log_d x}{\log_d b}$
- Examples of usage, coming up with recurrences

Questions

- log:remember that $\log_b x = \dfrac{\log_d x}{\log_d b}$
- Examples of usage, coming up with recurrences
- Is Master theorem tight?

Questions

- log:remember that $\log_b x = \frac{\log_d x}{\log_d b}$
- Examples of usage, coming up with recurrences
- Is Master theorem tight?
- Can d be 0?

Questions

- log:remember that $\log_b x = \frac{\log_d x}{\log_d b}$
- Examples of usage, coming up with recurrences
- Is Master theorem tight?
- Can d be 0?
- Why is Master theorem true? (intuition, proof)

Big Theta

- For each function $f(x)$ find a function $g(x)$ such that $f(x)$ is $\theta(g(x))$.

Big Theta

- For each function $f(x)$ find a function $g(x)$ such that $f(x)$ is $\theta(g(x))$.
  1. $f(x) = 10$
  2. $f(x) = 3x + 7$
  3. $f(x) = x^2 + x + 1$
  4. $f(x) = 5 \log x$
  5. $f(x) = floor(x)$
  6. $f(x) = ceiling(x/2)$

Big Theta

- For each function $f(x)$ find a function $g(x)$ such that $f(x)$ is $\theta(g(x))$.
  1. $f(x) = 10$

Big Theta

- For each function $f(x)$ find a function $g(x)$ such that $f(x)$ is $\theta(g(x))$.
  1. $f(x) = 10 \Rightarrow g(x) = 1$.
  2. $f(x) = 3x + 7$

Big Theta

- For each function $f(x)$ find a function $g(x)$ such that $f(x)$ is $\theta(g(x))$.
  1. $f(x) = 10 \Rightarrow g(x) = 1$.
  2. $f(x) = 3x + 7 \Rightarrow g(x) = x$.
  3. $f(x) = x^2 + x + 1$

Big Theta

- For each function $f(x)$ find a function $g(x)$ such that $f(x)$ is $\theta(g(x))$.
  1. $f(x) = 10 \Rightarrow g(x) = 1$.
  2. $f(x) = 3x + 7 \Rightarrow g(x) = x$.
  3. $f(x) = x^2 + x + 1 \Rightarrow g(x) = x^2$.
  4. $f(x) = 5 \log x$

Big Theta

- For each function $f(x)$ find a function $g(x)$ such that $f(x)$ is $\theta(g(x))$.
  1. $f(x) = 10 \Rightarrow g(x) = 1$.
  2. $f(x) = 3x + 7 \Rightarrow g(x) = x$.
  3. $f(x) = x^2 + x + 1 \Rightarrow g(x) = x^2$.
  4. $f(x) = 5 \log x \Rightarrow g(x) = \log x$.
  5. $f(x) = floor(x)$

Big Theta

- For each function $f(x)$ find a function $g(x)$ such that $f(x)$ is $\theta(g(x))$.
  1. $f(x) = 10 \Rightarrow g(x) = 1$.
  2. $f(x) = 3x + 7 \Rightarrow g(x) = x$.
  3. $f(x) = x^2 + x + 1 \Rightarrow g(x) = x^2$.
  4. $f(x) = 5 \log x \Rightarrow g(x) = \log x$.
  5. $f(x) = floor(x) \Rightarrow g(x) = x$.
  6. $f(x) = ceiling(x/2)$

Big Theta

- For each function $f(x)$ find a function $g(x)$ such that $f(x)$ is $\theta(g(x))$.
  1. $f(x) = 10 \Rightarrow g(x) = 1$.
  2. $f(x) = 3x + 7 \Rightarrow g(x) = x$.
  3. $f(x) = x^2 + x + 1 \Rightarrow g(x) = x^2$.
  4. $f(x) = 5 \log x \Rightarrow g(x) = \log x$.
  5. $f(x) = floor(x) \Rightarrow g(x) = x$.
  6. $f(x) = ceiling(x/2) \Rightarrow g(x) = x$.

Quiz

- Prove if $f(x)$ is $O(g(x))$, then $g(x)$ is $\Omega(f(x))$

Proving Master Theorem

f increasing function with $f(n) = af(n/b) + cn^d$, $a \geq 1, b \in \mathbb{N}, c, d > 0$.

- Show that if $a = b^d$ and n a power of b, then $f(n) = f(1)n^d + cn^d \log_b n$.

Proving Master Theorem

f increasing function with $f(n) = af(n/b) + cn^d$, $a \geq 1, b \in \mathbb{N}, c, d > 0$.

- Show that if $a = b^d$ and n a power of b, then $f(n) = f(1)n^d + cn^d \log_b n$.
- Show that if $a = b^d$, then $f(n)$ is $O(n^d \log n)$.

Applying Master Theorem

- Determine Big-O for the following:

1. $T(n) = 4T(n/5) + 6n^2$

Applying Master Theorem

- Determine Big-O for the following:

1. $T(n) = 4T(n/5) + 6n^2$
2. $T(n) = 4T(\frac{n}{2}) + n^2$

Applying Master Theorem

- Determine Big-O for the following:

1. $T(n) = 4T(n/5) + 6n^2$
2. $T(n) = 4T(\frac{n}{2}) + n^2$
3. $T(n) = 8T(n/2) + 6n + 7\log n$

Applying Master Theorem

- Determine Big-O for the following:

1. $T(n) = 4T(n/5) + 6n^2$
2. $T(n) = 4T(\frac{n}{2}) + n^2$
3. $T(n) = 8T(n/2) + 6n + 7\log n$
   Answer: $7\log n = O(n) \Rightarrow 7\log n \leq Cn$

Applying Master Theorem

- Determine Big-O for the following:

1. $T(n) = 4T(n/5) + 6n^2$

2. $T(n) = 4T(\frac{n}{2}) + n^2$

3. $T(n) = 8T(n/2) + 6n + 7\log n$
   Answer: $7\log n = O(n) \Rightarrow 7\log n \leq Cn$
   So, $T(n) = 8T(n/2) + 6n + 7\log n \leq 8T(n/2) + 6n + Cn$

Applying Master Theorem

- Determine Big-O for the following:

1. $T(n) = 4T(n/5) + 6n^2$
2. $T(n) = 4T(\frac{n}{2}) + n^2$
3. $T(n) = 8T(n/2) + 6n + 7\log n$
   Answer: $7\log n = O(n) \Rightarrow 7\log n \leq Cn$
   So, $T(n) = 8T(n/2) + 6n + 7\log n \leq 8T(n/2) + 6n + Cn = 8T(n/2) + (6+C)n$

Applying Master Theorem

- Determine Big-O for the following:

1. $T(n) = 4T(n/5) + 6n^2$
2. $T(n) = 4T(\frac{n}{2}) + n^2$
3. $T(n) = 8T(n/2) + 6n + 7\log n$
   Answer: $7\log n = O(n) \Rightarrow 7\log n \leq Cn$
   So, $T(n) = 8T(n/2) + 6n + 7\log n \leq 8T(n/2) + 6n + Cn = 8T(n/2) + (6 + C)n$
   Use Master Theorem: $8 > 2^1 \Rightarrow T(n) = O(n^{\log_2 8}) = O(n^3)$