CS313H
Logic, Sets, and Functions: Honors
Fall 2012

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Are there any questions?

Logistics

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- More Big-O practice on last slides of this slide deck

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- Big-O vs. "order"

Binary Search

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- Would it help to do a ternary or quaternary search?

Multi-person Elections

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  What is it's Big-O runtime?
  - Hint: If you split the sequence in half (or off by one), note that a candidate could not have an overall majority without receiving a majority of votes in at least one of the 2 halves.

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$T(n) = O(n^0 \log n) = O(\log n)$ which is better than $O(n)$.

Nuts and Bolts

- You are given a collection of n bolts of different widths and n corresponding nuts. You are allowed to try a nut and bolt together, from which you can determine whether the nut is larger than the bolt, smaller than the bolt, or matches the bolt exactly. However, there is no way to compare two nuts together or two bolts together. Create and analyze the expected runtime of an efficent algorithm to match each bolt to its nut.

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Solution: Randomly select a nut and traverse all bolts to find its match. Meanwhile, partition all bolts into two sets. One contains all bolts smaller than this nut and the other contains all bolts larger than this nut. Then after finding the matched bolt, use this bolt to do same partition for all nuts. These two partitions can be done in $2n$ comparisons. Then we need to deal with two sets, each of size $\frac{n}{2}$ (on average). Thus we get the recurrence relation below:

$$T(n) = 2T(\frac{n}{2}) + 2n$$

By Master Theorem, we have $T(n) = O(n \log n)$.

More Big-O practice

- Let a be any positive number. Show that $a^n = O(n!)$.

Solution

$$a^n = \underbrace{a \times a...a}_{n} \quad \text{and} \quad n! = n \times (n-1)...2 \times 1$$

When $a \leq 1$, we have $C = 1, k = 1$. When $a > 1$, let $k = 2a^2$, when $n > k$ we have $\frac{n}{2} > a^2$ and

$$\begin{aligned} n! &= n \times (n-1)...\frac{n}{2} \times (\frac{n}{2} - 1)... \times 1 \\ &> \underbrace{a^2 \times a^2...a^2}_{\frac{n}{2}} \times (\frac{n}{2} - 1)... \times 1 \\ &> (a^2)^{\frac{n}{2}} \\ &= a^n \end{aligned}$$

Thus we have $C = 1$, $k = \max(1, 2a^2)$ such that for all $x > k$, $a^n < Cn!$. So we have $a^n = O(n!)$. Proof completed.