CS313H
Logic, Sets, and Functions: Honors
Fall 2012

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Good Morning, Colleagues

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- How do we convert everything to CNF and DNF? Why do we care?
  - (answered in discussion)

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  - (answered in discussion)
- How do we write “There exists one and only one” (and its negation)?
  - (answered on piazza)

Logistics

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Some important concepts

- Multiple ways of converting the same English sentence to logic
- s.t. = "such that"
- Dogs and collars problem

Translate these statements into English:

1. $\forall x[(H(x) \land \neg \exists y M(x,y)) \rightarrow U(x)]$
   where H(x) = "x is a man", M(x,y) = "x is married to y", U(x) = "x is unhappy".

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2. $\exists z(P(z, Jake) \land S(z, Alex) \land W(Alex))$
   P(z,x) = "z is a parent of x", S(z,y) = "z and y are siblings", W(y) = "y is a woman".

3. $\forall n((P(n) \land n > 2) \rightarrow \neg\exists a, b, c(P(a) \land P(b) \land P(c) \land (a^n + b^n = c^n)))$
   where P(n) = "n is a positive integer".

Translate the following statements into logical notation

No new predicates (just use common mathematical symbols), where the domain is natural numbers.

1. x is a perfect square.

2. x is a multiple of y.

3. p is prime.

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1. x is a perfect square.
   $\exists y (x = y^2)$

2. x is a multiple of y.
   $\exists z (x = yz)$

3. p is prime.
   $(p \in \mathbb{Z}) \wedge (p > 1) \wedge \neg \exists x, y(x < p \wedge y < p \wedge (xy = p))$

Domains

How does the choice of domain for the following quantified statements affect whether each statement is true or false? The domains to pick from are $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$.

1. $\forall x \exists y (2x - y = 0)$

2. $\exists y \forall x (2x - y = 0)$

3. $\forall x \exists y (x - 2y = 0)$

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1. $\forall x \exists y (2x - y = 0)$
2. $\exists y \forall x (2x - y = 0)$
3. $\forall x \exists y (x - 2y = 0)$
4. $\forall x (x < 10 \rightarrow \forall y (y < x \rightarrow y < 9))$
5. $\exists x \exists y (x + y = 100)$
6. $\forall x \exists y (y > x \wedge \exists z (y + z = 100))$

True or False?

1. Domain: all real numbers
   $P(x, y)$: $x + y = 0$
   Predicate 1: $\forall x \exists y P(x, y)$
   Predicate 2: $\exists x \forall y P(x, y)$

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   $P(x, y)$: $x + y = 0$
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2. Domain: all rational numbers
   Predicate : $\forall x \exists y (x < y \wedge \neg \exists z (x < z \wedge z < y))$
   What if the domain is all integers?

Quiz: True or False?

- If P(x) = "x is prime"
- Q(x) = "x is even"
- the domain is the natural numbers

1. $P(5) \wedge Q(10) \wedge \neg Q(5) \wedge \neg P(4)$
2. $(\forall x P(x)) \rightarrow Q(4)$
3. $\neg \exists x, y(P(x) \wedge P(y) \wedge P(x+y))$
4. $\exists x(P(x) \wedge Q(x) \wedge \forall y((P(y) \wedge Q(y)) \rightarrow x = y))$
5. $\forall x(\neg P(x) \rightarrow Q(x))$
6. $\forall x((x > 2 \wedge P(x)) \rightarrow \exists y(Q(y) \wedge x = y + 1))$

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