Good Morning, Colleagues
Good Morning, Colleagues

Are there any questions?
Good Morning, Colleagues

Are there any questions?

- How do we convert everything to CNF and DNF? Why do we care?
  - (answered in discussion)
Good Morning, Colleagues

Are there any questions?

- How do we convert everything to CNF and DNF? Why do we care?
  - (answered in discussion)
- How do we write “There exists one and only one” (and its negation)?
  - (answered on piazza)
Logistics

- Office hours - try to let us know in advance if you’re coming
Logistics

- Office hours - try to let us know in advance if you’re coming
- Keep posting on piazza
Logistics

- Office hours - try to let us know in advance if you’re coming

- Keep posting on piazza

- First homework due at start of class
Some important concepts

- Multiple ways of converting the same English sentence to logic
- s.t. = “such that”
- Dogs and collars problem
Translate these statements into English:

1. \( \forall x [(H(x) \land \neg \exists y M(x, y)) \rightarrow U(x)] \)
   where \( H(x) = \) "x is a man" , \( M(x,y) = \) "x is married to y" ,
   \( U(x) = \) "x is unhappy". 
Translate these statements into English:

1. \( \forall x[(H(x) \land \neg \exists y M(x, y)) \rightarrow U(x)] \)
   where \( H(x) = "x \text{ is a man}" \), \( M(x,y) = "x \text{ is married to } y" \),
   \( U(x) = "x \text{ is unhappy}" \).

2. \( \exists z(P(z, Jake) \land S(z, Alex) \land W(Alex)) \)
   \( P(z,x) = "z \text{ is a parent of } x" \), \( S(z,y) = "z \text{ and } y \text{ are siblings}" \),
   \( W(y) = "y \text{ is a woman}" \).

3. \( \forall n((P(n) \land n > 2) \rightarrow \neg \exists a, b, c(P(a) \land P(b) \land P(c) \land (a^n + b^n = c^n))) \)
   where \( P(n) = "n \text{ is a positive integer}" \).
Translate the following statements into logical notation

No new predicates (just use common mathematical symbols), where the domain is natural numbers.

1. $x$ is a perfect square.

2. $x$ is a multiple of $y$.

3. $p$ is prime.
Translate the following statements into logical notation

No new predicates (just use common mathematical symbols), where the domain is natural numbers.

1. x is a perfect square.
   \[ \exists y (x = y^2) \]

2. x is a multiple of y.
   \[ \exists z (x = yz) \]

3. p is prime.
   \[ (p \in \mathbb{Z}) \land (p > 1) \land \neg \exists x, y (x < p \land y < p \land (xy = p)) \]
Domains

How does the choice of domain for the following quantified statements affect whether each statement is true or false? The domains to pick from are \( \mathbb{N} \), \( \mathbb{Z} \), \( \mathbb{Q} \) and \( \mathbb{R} \).

1. \( \forall x \exists y \left( 2x - y = 0 \right) \)

2. \( \exists y \forall x \left( 2x - y = 0 \right) \)

3. \( \forall x \exists y \left( x - 2y = 0 \right) \)
Domains

How does the choice of domain for the following quantified statements affect whether each statement is true or false? The domains to pick from are $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$.

1. $\forall x \exists y (2x - y = 0)$

2. $\exists y \forall x (2x - y = 0)$

3. $\forall x \exists y (x - 2y = 0)$

4. $\forall x (x < 10 \rightarrow \forall y (y < x \rightarrow y < 9))$

5. $\exists x \exists y (x + y = 100)$

6. $\forall x \exists y (y > x \land \exists z (y + z = 100))$
True or False?

1. Domain: all real numbers
   \[ P(x, y): x + y = 0 \]
   Predicate 1: \( \forall x \exists y P(x, y) \)
   Predicate 2: \( \exists x \forall y P(x, y) \)
True or False?

1. Domain: all real numbers
   \( P(x, y) \): \( x + y = 0 \)
   Predicate 1: \( \forall x \exists y P(x, y) \)
   Predicate 2: \( \exists x \forall y P(x, y) \)

2. Domain: all rational numbers
   Predicate: \( \forall x \exists y (x < y \land \neg \exists z (x < z \land z < y)) \)
   What if the domain is all integers?
Quiz: True or False?

- If $P(x)$ = “$x$ is prime”
- $Q(x)$ = “$x$ is even”
- the domain is the natural numbers

1. $P(5) \land Q(10) \land \neg Q(5) \land \neg P(4)$
2. $(\forall x P(x)) \rightarrow Q(4)$
3. $\neg \exists x, y (P(x) \land P(y) \land P(x + y))$
4. $\exists x (P(x) \land Q(x) \land \forall y ((P(y) \land Q(y)) \rightarrow x = y))$
5. $\forall x (\neg P(x) \rightarrow Q(x))$
6. $\forall x ((x > 2 \land P(x)) \rightarrow \exists y (Q(y) \land x = y + 1))$
Assignments for Tuesday

- First homework **due at start of class**
- Modules 4,5 with associated readings
Assignments for Tuesday

- First homework **due at start of class**
- Modules 4,5 with associated readings