CS313H
Logic, Sets, and Functions: Honors
Fall 2012

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Good Morning, Colleagues

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Are there any questions?

Logistics

- Keeping up and posting on piazza is **required**

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- Second homework **due at start of class**

Some questions

- Is this CS or math?

Prove that. . .

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Prove by Contradiction

- There are no positive integer solutions to equation $x^2 - y^2 = 1$

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  6. So, $A^2 < 1/100 \Rightarrow A < 1/10$.

Prove that. . .

- Show that there is no rational number r for which r^3 + r + 1 = 0

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