CS313H
Logic, Sets, and Functions: Honors
Fall 2012

Prof: Peter Stone
TA: Jacob Schrum
Proctor: Sudheesh Katkam

Department of Computer Science
The University of Texas at Austin

Odd snowball fight

- An odd number of people stand on a football field. No two people are the same distance from each other as any other two people. When I shout “go”, everyone throws a snowball at his/her nearest neighbor, hitting this person. Prove that at least one person is not hit by a snowball (a “survivor”).

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Are there any questions?

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- Base case: n=1 (3 people)

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Tricky problem from Piazza

- Prove that when $n \geq 1, a_i \in \mathbb{R}, a_i > 0$, if $a_1 \times a_2 \times ... \times a_n = 1$, then

$$(1 + a_1)(1 + a_2)...(1 + a_n) \geq 2^n$$

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  2. $n \geq 10 \Rightarrow n - 2 \geq 8$
  3. Since $n \geq n - 2 \geq 8$, $P(n-2)$ is true
  4. So, $n + 1 = (n - 2) + 3 = 3i + 5j + 3 = 3(i + 1) + 5j$

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  1. $n + 1 = n - 2 + 3$
  2. $n \geq 10 \Rightarrow n - 2 \geq 8$
  3. Since $n \geq n - 2 \geq 8$, $P(n - 2)$ is true
  4. So, $n + 1 = (n - 2) + 3 = 3i + 5j + 3 = 3(i + 1) + 5j$
  5. So, $n + 1$ cents can be paid with 3 and 5 cent stamps.

Tiling a holey checkerboard

- Let n be a positive integer. Show that every $2^n \times 2^n$ checkerboard with one square removed can be tiled using right triominoes (pieces shaped like the letter "L") that cover three squares of the board.

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- Let n be a positive integer. Show that every $2^n \times 2^n$ checkerboard with one square removed can be tiled using right triominoes (pieces shaped like the letter "L") that cover three squares of the board.

- (see proof in the book)

Strong induction

- Which amounts of money can be formed using just \$2 and \$5 bills?

More tiling

- Prove: When $n > 1$, Assume we have three kinds of tiles: 1 by 2 tiles, 2 by 1 tiles and 2 by 2 tiles. Prove given a n by 2 board, there are

$$\frac{2^{n+1} + (-1)^n}{3}$$

ways to fill it using these three kinds of tiles.

Prove:

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- $(n^2 - 1)$ is divisible by 8 whenever n is an odd positive integer

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