CS313H
Logic, Sets, and Functions: Honors
Fall 2012

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Challenge

- Prove that for any non-empty set A, there does not exist a bijective function from A to P(A) where P(A) is power set of A (remember that A could be infinite).

Good Morning, Colleagues

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Are there any questions?

Logistics

- Start/keep reviewing everything we've done

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  - Different types of infinity

Some important concepts

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- Sets vs. tuples
- Cartesian product: deck of cards, plane
- injection, surjection, bijection

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- $P(A \cap B) = P(A) \cap P(B)$ (use previous problem's result)

Prove that. . .

- $A \subseteq B$ iff $P(A) \subseteq P(B)$.

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- $(A \bigcup B) \times C = (A \times C) \bigcup (B \times C)$.

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  1. Let $(g \circ f)(x) = (g \circ f)(y)$ for $x, y \in A$
  2. $\equiv g(f(x)) = g(f(y))$ {Def $\circ$}
  3. $\Rightarrow f(x) = f(y)$ {g is injective}
  4. $\Rightarrow x = y$ {f is injective}

  Therefore $g \circ f$ is injective.

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  1. Pick arbitrary $c \in C$.

Prove that...

- Suppose $f : A \to B$ and $g : B \to C$. Prove that if f and g are injective, then $g \circ f$ is injective.
  1. Let $(g \circ f)(x) = (g \circ f)(y)$ for $x, y \in A$
  2. $\equiv g(f(x)) = g(f(y))$ {Def $\circ$}
  3. $\Rightarrow f(x) = f(y)$ {g is injective}
  4. $\Rightarrow x = y$ {f is injective}

  Therefore $g \circ f$ is injective.
- Suppose $f : A \to B$ and $g : B \to C$. Prove that if f and g are surjective, then $g \circ f$ is surjective.
  1. Pick arbitrary $c \in C$.
  2. Since g is onto, there is a $b \in B$ s.t. $g(b) = c$.
  3. Since $b \in B$, and f is onto, there is an $a \in A$ s.t. $f(a) = b$.
  4. $(g \circ f)(a) = g(f(a)) = g(b) = c$.

  Therefore $g \circ f$ is surjective.

Assignments for Thursday

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