Good Morning, Colleagues
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Are there any questions?
Logistics

• Midterm 1, Tuesday
  – Handwritten notes allowed
  – No book, nothing printed, nothing electronic
  – Be on time!
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- 2 modules due Thursday after exam
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  - First is mainly definitions - may want to do it this week
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- HW4 review
A Bijection That Works

Use C-B-S to prove that \(|[0, 1)| = |(0, 1)|\)
A Bijection That Works

Use C-B-S to prove that $|[0, 1)| = |(0, 1)|$

$$f(x) = \begin{cases} 
1/2 & \text{if } x = 0 \\
\frac{x}{1 + x} & \text{if } \exists n \in \mathbb{N}[x = 1/n] \\
x & \text{otherwise}
\end{cases}$$

Pete Stone
Satisfiable or unsatisfiable?

\[ \neg (X_1 \lor \neg (X_2 \land X_3) \lor (\neg X_1 \land X_3 \land \neg (X_1 \lor \neg X_2))) \]
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Unsatisfiable.

Some simplification first will make this easier to see:
\[ \neg(X_1 \lor \neg(X_2 \land X_3) \lor (\neg X_1 \land X_3 \land \neg(X_1 \lor \neg X_2))) \] (original)
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\[ \equiv \neg X_1 \land X_2 \land X_3 \land \neg(\neg X_1 \land X_3 \land \neg(X_1 \lor \neg X_2)) \text{ (De Morgan, double neg.)} \]
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  $\equiv \neg X_1 \land X_2 \land X_3 \land (X_1 \lor \neg X_3 \lor X_1 \lor \neg X_2)$ (De Morgan, double neg.)
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Translation

1. Domain: all human beings
   \( P(x) \): \( x \) has blue eyes
   \( Q(x) \): \( x \) has black eyes
   Statement: There exist people with blue eyes and with black eyes, but one cannot have blue and black eyes at the same time.

2. Domain: all UT student
   \( P(x) \): \( x \) is a computer science student
   \( Q(x) \): \( x \) must take 313
   Statement: \( \forall x (P(x) \rightarrow Q(x)) \)
Translation

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   Answer: $\exists x P(x) \land \exists x Q(x) \land \forall x \neg(P(x) \land Q(x))$
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1. Domain: all human beings
   \[ P(x): x \text{ has blue eyes} \]
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   Answer: \[ \exists x P(x) \land \exists x Q(x) \land \forall x \neg (P(x) \land Q(x)) \]

2. Domain: all UT student
   \[ P(x): x \text{ is a computer science student} \]
   \[ Q(x): x \text{ must take 313} \]
   Statement: \[ \forall x (P(x) \rightarrow Q(x)) \]
   Answer: All computer science students in UT must take 313
Pair up activity

- Suppose $B$ and $C$ are disjoint. Prove $(A \times B) \cap (A \times C) = \emptyset$. 
Pair up activity

• Suppose $B$ and $C$ are disjoint. Prove $(A \times B) \cap (A \times C) = \emptyset$.

• Sponge: Suppose $A \subseteq B \subseteq C$. Prove $C - B \subseteq C - A$. 
Induction

Prove: When \( n > 1 \), Assume we have three kinds of tiles: 1 by 2 tiles, 2 by 1 tiles and 2 by 2 tiles. Prove given a \( n \) by 2 board, there are

\[
\frac{2^{n+1} + (-1)^n}{3}
\]

ways to fill it using these three kinds of tiles.
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Assignments for Thursday

- Modules 10 and 11