CS313H
Logic, Sets, and Functions: Honors
Fall 2012

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- Graph of degree k colorable with $k+1$ colors
  - Clever predicate!

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- New unit: graph theory and counting

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2. 12 couples go to a party and everyone shakes hands with everyone except for their spouse.

3. Three groups of people go to a party. No one shakes hands with anyone from the group they came with but they all shake hands with everyone else. The sizes of the three groups are 4, 6 and 10.

What's the induced subgraph?

- Vertices $\{v_1, v_2, v_3\}$ of graph
  $G = (\{v_1, v_2, v_3, v_4\}, \{(v_1, v_2), (v_2, v_4), (v_3, v_4), (v_2, v_3)\})$

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  Answer: $(\{v_1, v_2, v_3\}, \{(v_1, v_2), (v_2, v_3)\})$.

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  4. Self-loops are not allowed, so this is a contradiction.

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2. A simple graph with 8 vertices, whose degrees are 0, 1, 2, 3, 4, 5, 6, 7.

3. A simple graph with degrees 1, 2, 2, 3.

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1. A simple graph with 6 vertices, whose degrees are 2, 2, 2, 3, 4, 4.
   It is not possible to have one vertex of odd degree.

2. A simple graph with 8 vertices, whose degrees are 0, 1, 2, 3, 4, 5, 6, 7.
   It is not possible to have a vertex of degree 7 and a vertex of degree 0 in this graph.

3. A simple graph with degrees 1, 2, 2, 3.
   Possible: v_1, v_2, v_3, v_4. Edges: $(v_1, v_2), (v_1, v_3), (v_1, v_4), (v_2, v_3)$.

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  Thus the vertices can have at most n-1 different degrees.
  Therefore at least 2 must have the same degree.

Chromatic Number

Find the chromatic number k, and define a valid k-coloring for each graph.

- $G = (\{v_1, v_2, v_3, v_4\}, \{(v_1, v_2), (v_1, v_3), (v_2, v_3), (v_2, v_4), (v_3, v_4)\})$

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  Chromatic number is 3, and a valid 3-coloring is v_1 and v_4 RED, v_2 BLUE, and v_3 GREEN.

Prove by Induction

- For $n > 0$, suppose n star graphs are linked in a chain, such that there is one edge connecting some vertex in the i^{th} graph with some vertex in the $(i+1)^{th}$ graph for all i where $0 < i < n$. Prove that the resulting graph is 2-colorable.

Scheduling

- The Math Department has 6 committees that meet once a month. How many different meeting times must be used to guarantee that no one is scheduled to be at 2 meetings at the same time, if committees and their members are: C1 = {Allen, Brooks, Marg}, C2 = {Brooks, Jones, Morton}, C3 = {Allen, Marg, Morton}, C4 = {Jones, Marg, Morton}, C5 = {Allen, Brooks}, C6 = {Brooks, Marg, Morton}.

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  Ans:

  We can draw a graph with C1 to C6 as vertices and an edge between the vertices if they share common elements. The answer is again the chromatic number of the graph - 5. Only C4 and C5 do not share any common elements.

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- Prove that for any even number n, there exists a graph with n vertices that has these properties.
  Define: $V(G, n) \equiv$ "graph G has n vertices"
  Formally, prove: $\forall k > 0, \exists G[V(G, 2k) \wedge D(G) \wedge I(G) \wedge O(G)]$.

Assignments for Tuesday

- Modules 12 and 13