

CS313H
Logic, Sets, and Functions: Honors
Fall 2012

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Warm ups

- How many sequences of 7 digits have at least one repeating digit?
- How many ways are there to arrange the letters in the word "SYSTEMS"?
- How many hands of 5 cards have at least 3 aces?

Good Morning, Colleagues

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Are there any questions?

Logistics

- Class survey

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 - Don't like 9pm quest deadline

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- Homework 6 due Thursday (+ Module 18)
 - Homework 7 due following Tuesday
- Midterm on graph theory, counting, recurrences following Thursday

Important counting concepts

- Addition rule
- Inclusion/exclusion principle
- Correspondence principle
- Product rule
- Number of subsets of an n element set: 2^n
- Number of permutations of n distinct objects: $n!$
- Number of subsets of size k from an n -element set: $\binom{n}{k}$
("n choose k") = $\frac{n!}{k!(n-k)!}$

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Counting Poker Hands

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- a **straight** is a sequence of 5 cards of consecutive rank
- a **flush** is a set of 5 cards with the same suit
- How many hands total? $\binom{52}{5} = 2,598,960$

How many? What are the odds?

Straight flush: straight and a flush

4 of a kind: 4 cards of the same rank

full house: 3 cards of one rank, two of another

flush: a flush but *not* a straight

straight: a straight but *not* a flush

3 of a kind: 3 cards of one rank, but not full house or 4 of a kind

2 pair: 2 cards of one rank, 2 of another rank, but *not* 4 of a kind or full house

pair: 2 cards of one rank, but not anything higher

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pair: $\frac{1,098,240}{2,598,960} = .4225 \approx 1 \text{ in } 2.4$