CS313H
Logic, Sets, and Functions: Honors
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Good Morning, Colleagues

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Are there any questions?

Logistics

- Have you figured out the poker hands?

Some questions

- Why does Pascal's triangle work?
- A few more facts

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- How many non negative integer solutions are there to $p + q + r < 11$?

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- (HARD) In a month with 30 days, a baseball team will play 45 games. It must also play at least one game on each day. Show that there will be a period of *consecutive* days where exactly 14 games are played.

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- Coefficient of $x_1^{r_1}x_2^{r_2}\cdots x_k^{r_k}$ in $(x_1+x_2+\cdots x_k)^n$ = $\frac{n!}{r_1!r_2!\cdots r_n!}$

Recurrences

- I can climb up stairs by taking either one stair or two stairs at a time. Let a_n be the number of ways I can climb n stairs. Find the recurrence relationship for a_n.

- Given recurrence relationship $a_n = 2a_{n-1} + 1$, $n \geq 2$ and initial condition $a_1 = 1$. Prove that when $n \geq 1$, $a_n = 2^n - 1$.