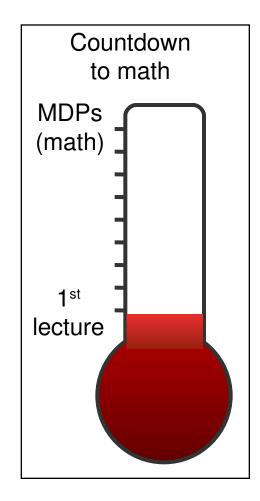
# Al Adjacent Fields

- Philosophy:
  - Logic, methods of reasoning
  - Mind as physical system
  - Foundations of learning, language, rationality
- Mathematics
  - Formal representation and proof
  - Algorithms, computation, (un)decidability, (in)tractability
  - Probability and statistics
- Psychology
  - Adaptation
  - Phenomena of perception and motor control
  - Experimental techniques (psychophysics, etc.)
- Economics: formal theory of rational decisions
- Linguistics: knowledge representation, grammar
- Neuroscience: physical substrate for mental activity
- Control theory:
  - homeostatic systems, stability
  - simple optimal agent designs

This slide deck courtesy of Dan Klein at UC Berkeley

### How Much of AI is Math?

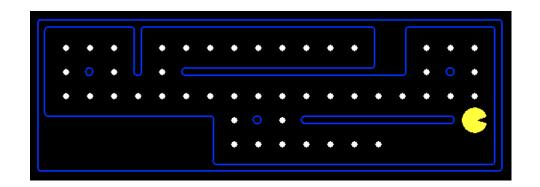
- A lot, but not right away
- Understanding probabilities will help you a great deal
- In later weeks, there will be many more equations

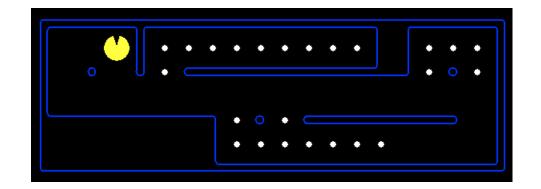


# Reflex Agents

#### Reflex agents:

- Choose action based on current percept (and maybe memory)
- May have memory or a model of the world's current state
- Do not consider the future consequences of their actions
- Consider how the world IS



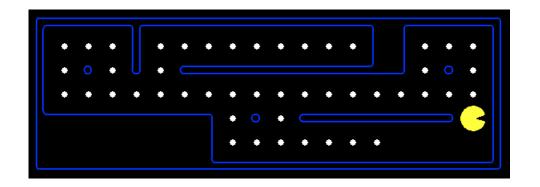


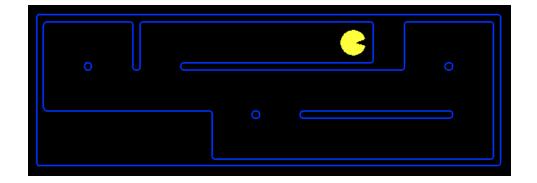
Can a reflex agent be rational?

# Goal Based Agents

#### Goal-based agents:

- Plan ahead
- Ask "what if"
- Decisions based on (hypothesized) consequences of actions
- Must have a model of how the world evolves in response to actions
- Consider how the world WOULD BE





#### Search Problems

- A search problem consists of:
  - A state space







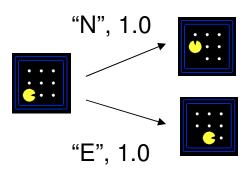






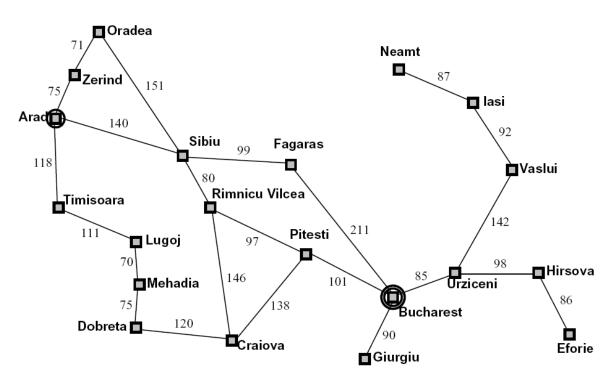


 A successor function (with actions, costs)



- A start state and a goal test
- A solution is a sequence of actions (a plan) which transforms the start state to a goal state

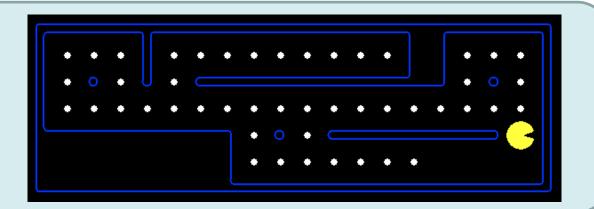
# Example: Romania



- State space:
  - Cities
- Successor function:
  - Roads: Go to adj city with cost = dist
- Start state:
  - Arad
- Goal test:
  - Is state == Bucharest?
- Solution?

# What's in a State Space?

The world state specifies every last detail of the environment



A search state keeps only the details needed (abstraction)

- Problem: Pathing
  - States: (x,y) location
  - Actions: NSEW
  - Successor: update location only
  - Goal test: is (x,y)=END

- Problem: Eat-All-Dots
  - States: {(x,y), dot booleans}
  - Actions: NSEW
  - Successor: update location and possibly a dot boolean
  - Goal test: dots all false

# State Space Sizes?

#### World state:

Agent positions: 120

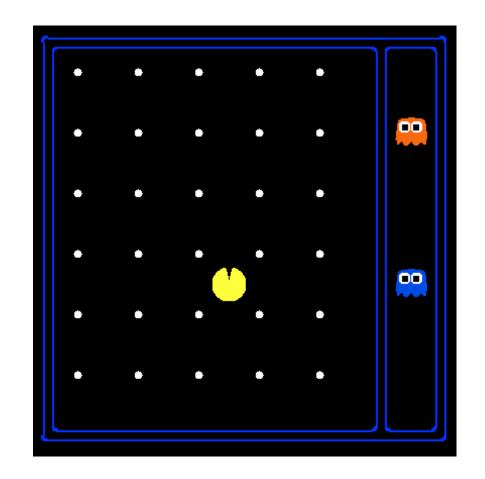
Food count: 30

Ghost positions: 12

Agent facing: NSEW

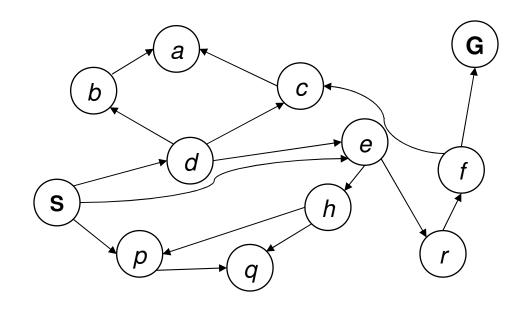
#### How many

- World states?
   120x(2<sup>30</sup>)x(12<sup>2</sup>)x4
- States for pathing?120
- States for eat-all-dots? 120x(2<sup>30</sup>)



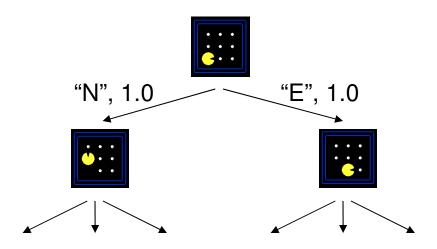
# State Space Graphs

- State space graph: A mathematical representation of a search problem
  - For every search problem, there's a corresponding state space graph
  - The successor function is represented by arcs
- We can rarely build this graph in memory (so we don't)



Ridiculously tiny search graph for a tiny search problem

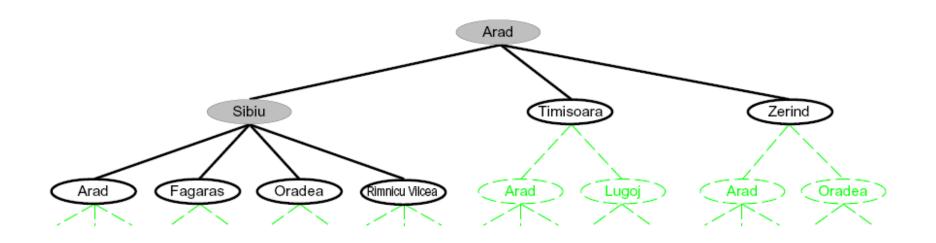
#### Search Trees



#### A search tree:

- This is a "what if" tree of plans and outcomes
- Start state at the root node
- Children correspond to successors
- Nodes contain states, correspond to PLANS to those states
- For most problems, we can never actually build the whole tree

#### Another Search Tree



#### Search:

- Expand out possible plans
- Maintain a fringe of unexpanded plans
- Try to expand as few tree nodes as possible

#### General Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure initialize the search tree using the initial state of problem loop do

if there are no candidates for expansion then return failure choose a leaf node for expansion according to strategy

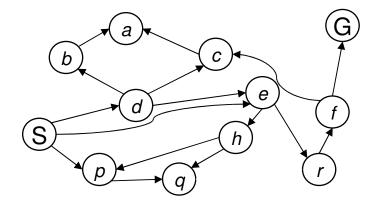
if the node contains a goal state then return the corresponding solution else expand the node and add the resulting nodes to the search tree end
```

- Important ideas:
  - Fringe
  - Expansion
  - Exploration strategy

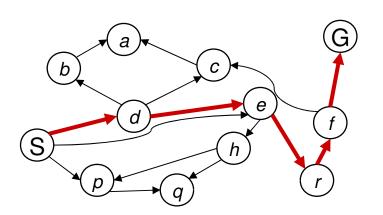
Detailed pseudocode is in the book!

Main question: which fringe nodes to explore?

# Example: Tree Search

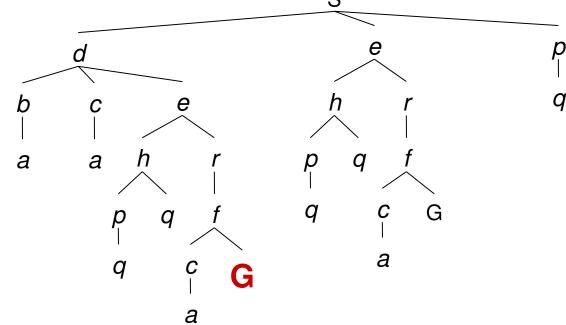


# State Graphs vs. Search Trees



Each NODE in in the search tree is an entire PATH in the problem graph.

We construct both on demand – and we construct as little as possible.



#### States vs. Nodes

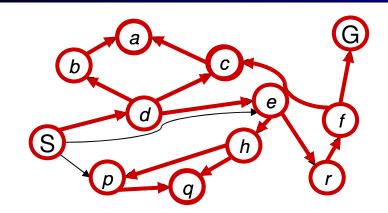
- Nodes in state space graphs are problem states
  - Represent an abstracted state of the world
  - Have successors, can be goal / non-goal, have multiple predecessors
- Nodes in search trees are plans
  - Represent a plan (sequence of actions) which results in the node's state
  - Have a problem state and one parent, a path length, a depth & a cost
  - The same problem state may be achieved by multiple search tree nodes

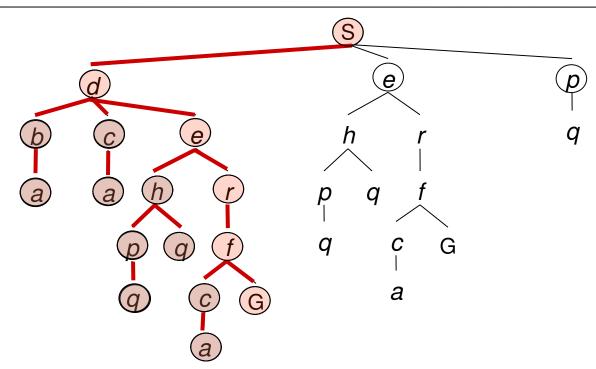
# Problem States Search Nodes Parent Action Depth 5

# Review: Depth First Search

Strategy: expand deepest node first

Implementation: Fringe is a LIFO stack

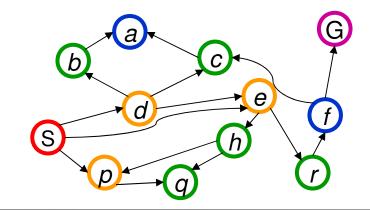


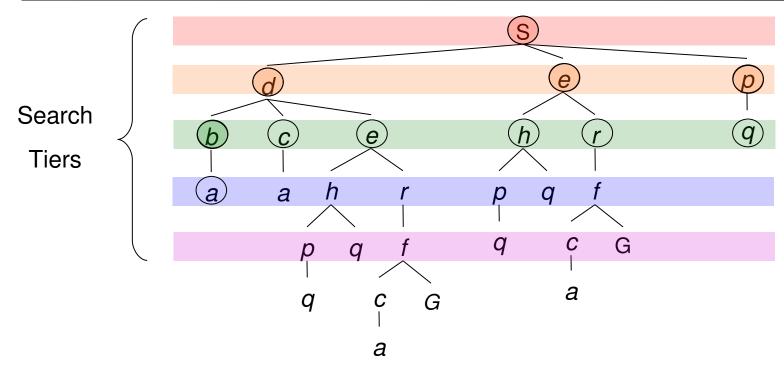


#### Review: Breadth First Search

Strategy: expand shallowest node first

Implementation: Fringe is a FIFO queue





## Search Algorithm Properties

Complete? Guaranteed to find a solution if one exists?

Optimal? Guaranteed to find the least cost path?

Time complexity?

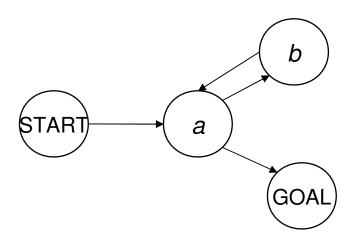
Space complexity?

#### Variables:

n	Number of states in the problem (huge)
b	The average branching factor B
	(the average number of successors)
$C^*$	Cost of least cost solution
S	Depth of the shallowest solution
m	Max depth of the search tree

## **DFS**

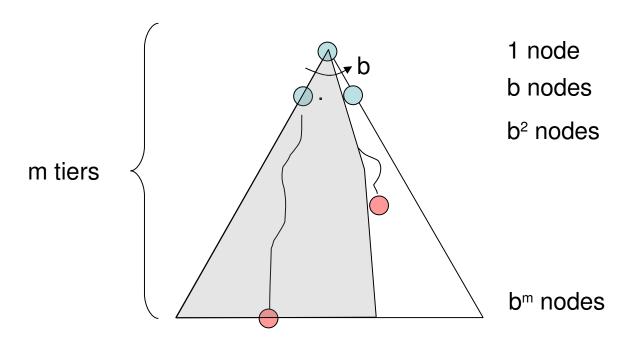
Algorithm		Complete	Optimal	Time	Space
DFS	Depth First Search	N	N	Infinite	Infinite



- Infinite paths make DFS incomplete...
- How can we fix this?

## **DFS**

With cycle checking, DFS is complete.\*

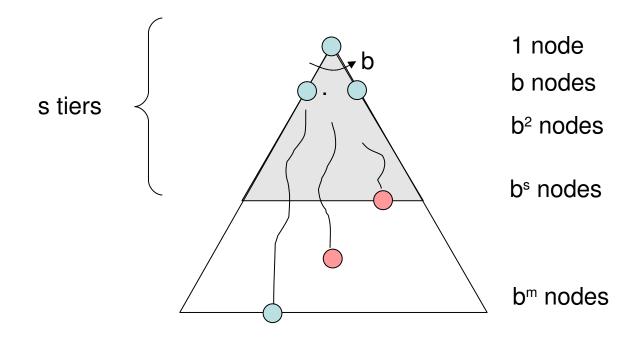


Algorith	m	Complete	Optimal	Time	Space
DFS	w/ Path Checking	Υ	Z	$O(b^{m+1})$	O(bm)

When is DFS optimal?

# **BFS**

Algorithm		Complete	Optimal	Time	Space
וטוט	w/ Path Checking	Y	N	$O(b^{m+1})$	O(bm)
BFS		Υ	N*	$O(b^{s+1})$	$O(b^s)$



When is BFS optimal?

# Comparisons

When will BFS outperform DFS?

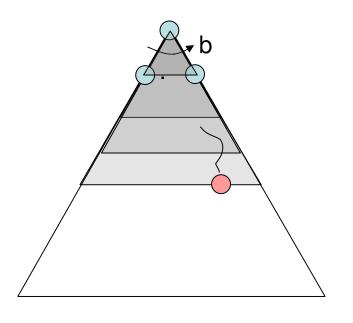
When will DFS outperform BFS?

# Iterative Deepening

Iterative deepening: BFS using DFS as a subroutine:

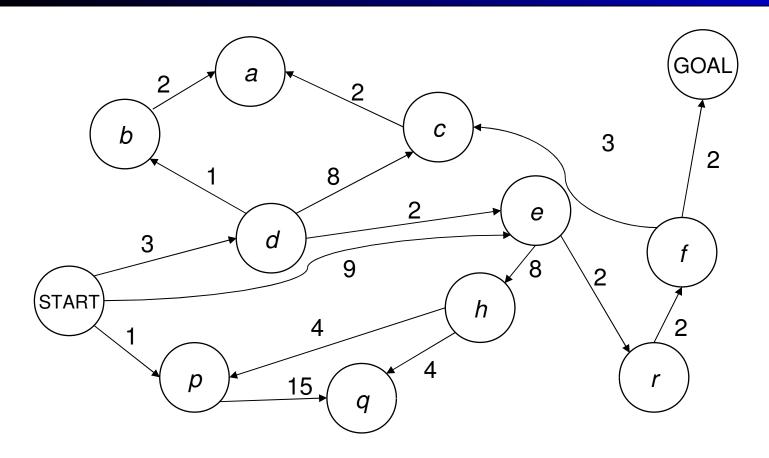
- 3. Do a DFS which only searches for paths of length 1 or less.
- 4. If "1" failed, do a DFS which only searches paths of length 2 or less.
- 5. If "2" failed, do a DFS which only searches paths of length 3 or less.

....and so on.



Algorithm		Complete	Optimal	Time	Space
	w/ Path Checking	Υ	N	$O(b^{m+1})$	O(bm)
BFS		Y	N*	$O(b^{s+1})$	$O(b^s)$
ID		Υ	N*	$O(b^{s+1})$	O(bs)

#### Costs on Actions



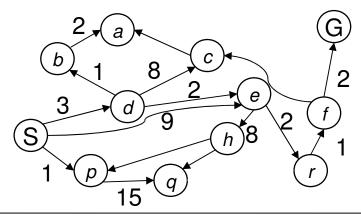
Notice that BFS finds the shortest path in terms of number of transitions. It does not find the least-cost path.

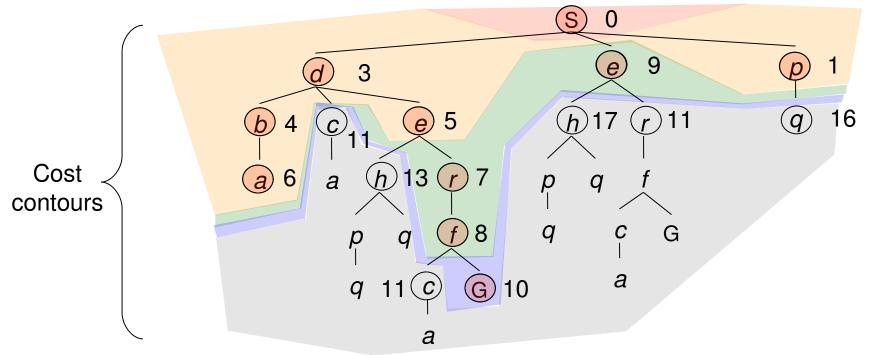
We will quickly cover an algorithm which does find the least-cost path.

## Uniform Cost Search

Expand cheapest node first:

Fringe is a priority queue (priority: cumulative cost)







# Priority Queue Refresher

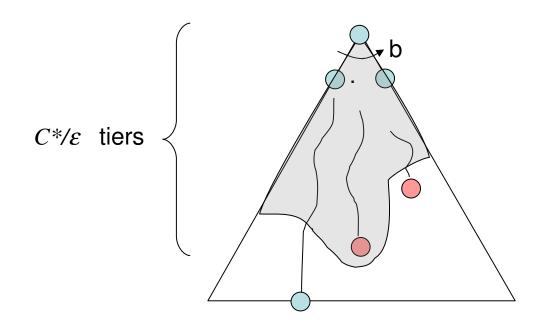
A priority queue is a data structure in which you can insert and retrieve (key, value) pairs with the following operations:

pq.push(key, value)	inserts (key, value) into the queue.
pq.pop()	returns the key with the lowest value, and removes it from the queue.

- You can decrease a key's priority by pushing it again
- Unlike a regular queue, insertions aren't constant time, usually O(log n)
- We'll need priority queues for cost-sensitive search methods

## Uniform Cost Search

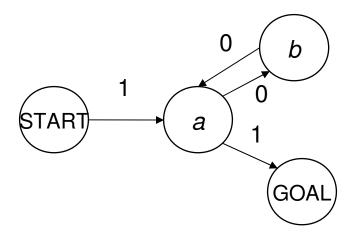
Algorithm		Complete	Optimal	Time	Space
DFS	w/ Path Checking	Υ	N	$O(b^{m+1})$	O(bm)
BFS		Υ	N	$O(b^{s+1})$	$O(b^s)$
UCS		Y*	Υ	$\mathrm{O}(b^{C*/arepsilon})$	$O(b^{C*/\varepsilon})$



\* UCS can fail if actions can get arbitrarily cheap

#### **Uniform Cost Search**

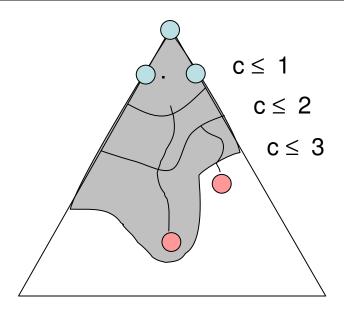
What will UCS do for this graph?

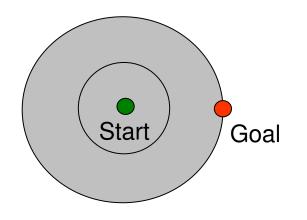


What does this mean for completeness?

#### **Uniform Cost Issues**

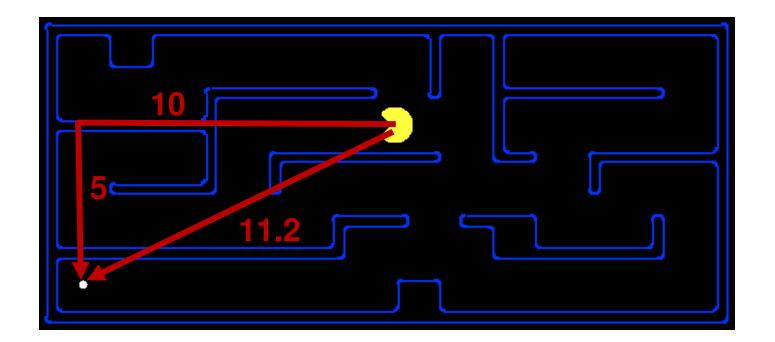
- Remember: explores increasing cost contours
- The good: UCS is complete and optimal!
- The bad:
  - Explores options in every "direction"
  - No information about goal location



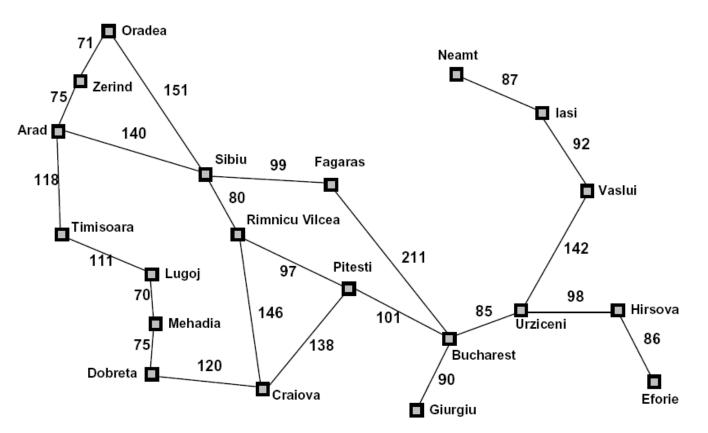


#### Search Heuristics

- Any estimate of how close a state is to a goal
- Designed for a particular search problem
- Examples: Manhattan distance, Euclidean distance



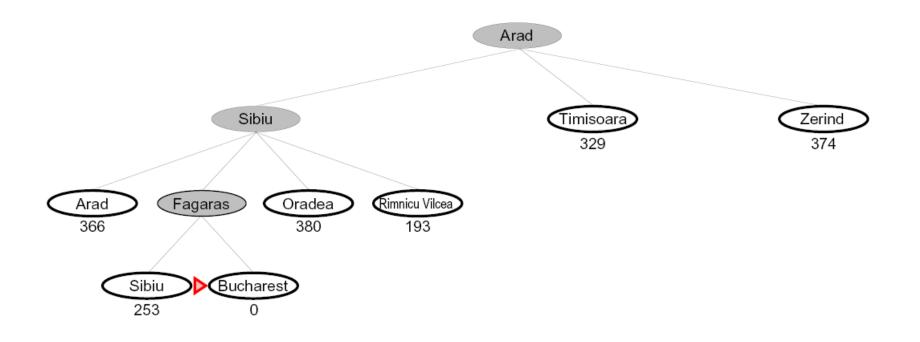
## Heuristics



Straight-line distance				
to Bucharest				
Arad	366			
Bucharest	0			
Craiova	160			
Dobreta	242			
Eforie	161			
Fagaras	178			
Giurgiu	77			
Hirsova	151			
Iasi	226			
Lugoj	244			
Mehadia	241			
Neamt	234			
Oradea	380			
Pitesti	98			
Rimnicu Vilcea	193			
Sibiu	253			
Timisoara	329			
Urziceni	80			
Vaslui	199			
Zerind	374			

# Best First / Greedy Search

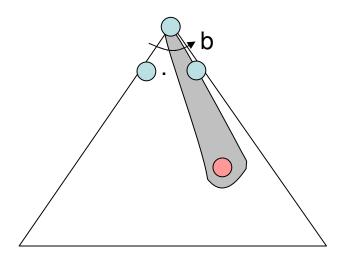
Expand the node that seems closest...

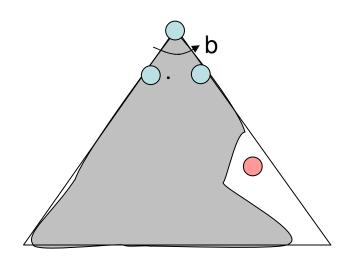


What can go wrong?

# Best First / Greedy Search

- A common case:
  - Best-first takes you straight to the (wrong) goal
- Worst-case: like a badlyguided DFS in the worst case
  - Can explore everything
  - Can get stuck in loops if no cycle checking
- Like DFS in completeness (finite states w/ cycle checking)





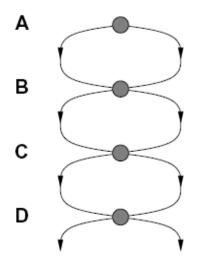
# Search Gone Wrong?

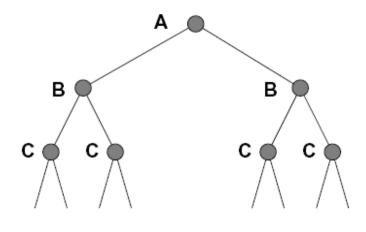




#### Extra Work?

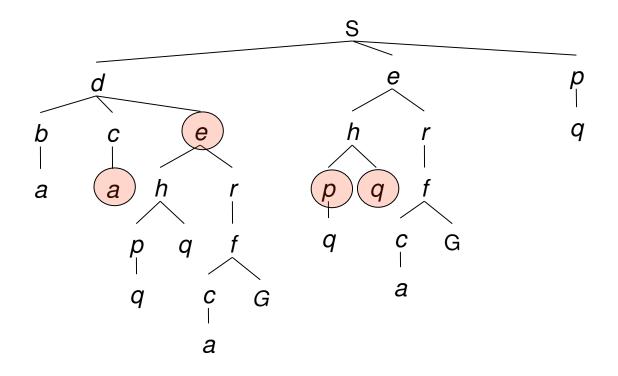
 Failure to detect repeated states can cause exponentially more work (why?)





# Graph Search

 In BFS, for example, we shouldn't bother expanding the circled nodes (why?)



# Graph Search

Very simple fix: never expand a state type twice

```
function GRAPH-SEARCH (problem, fringe) returns a solution, or failure
   closed \leftarrow an empty set
   fringe \leftarrow Insert(Make-Node(Initial-State[problem]), fringe)
   loop do
       if fringe is empty then return failure
       node \leftarrow \text{Remove-Front}(fringe)
       if Goal-Test(problem, State[node]) then return node
       if State [node] is not in closed then
            add STATE[node] to closed
            fringe \leftarrow InsertAll(Expand(node, problem), fringe)
   end
```

- Can this wreck completeness? Why or why not?
- How about optimality? Why or why not?

#### Some Hints

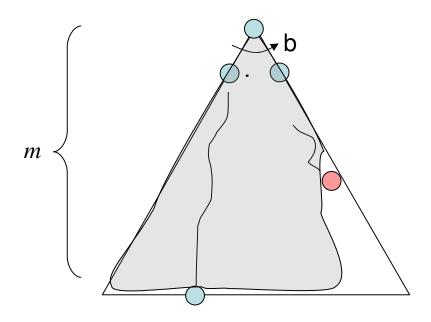
 Graph search is almost always better than tree search (when not?)

Implement your closed list as a dict or set!

 Nodes are conceptually paths, but better to represent with a state, cost, last action, and reference to the parent node

# Best First Greedy Search

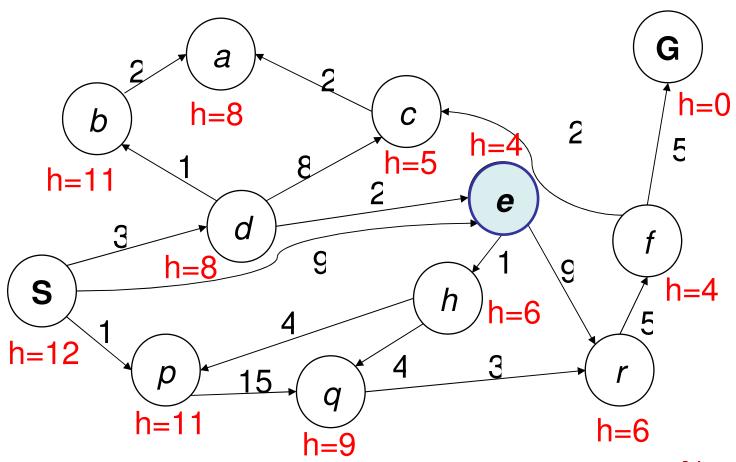
Algorithm	Complete	Optimal	Time	Space
Greedy Best-First Search	Y*	N	$O(b^m)$	$O(b^m)$



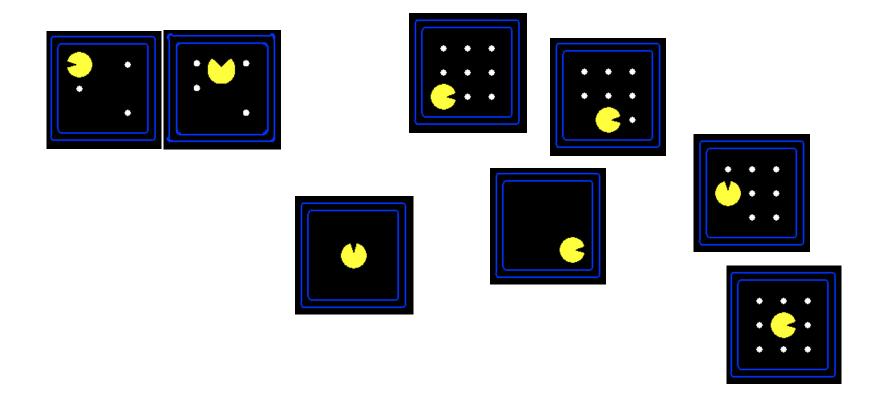
- What do we need to do to make it complete?
- Can we make it optimal? Next class!

# Best First / Greedy Search

Strategy: expand the closest node to the goal



[demo: greedy]



# Example: Tree Search

