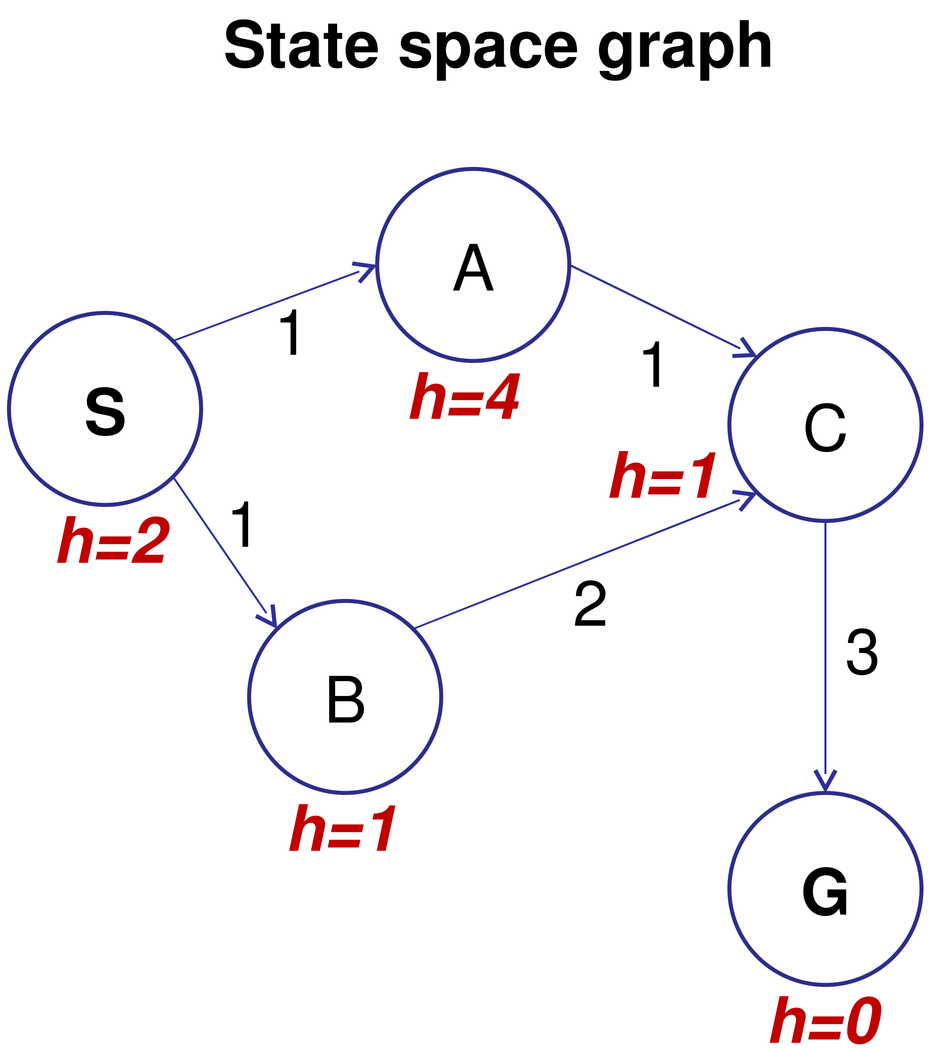
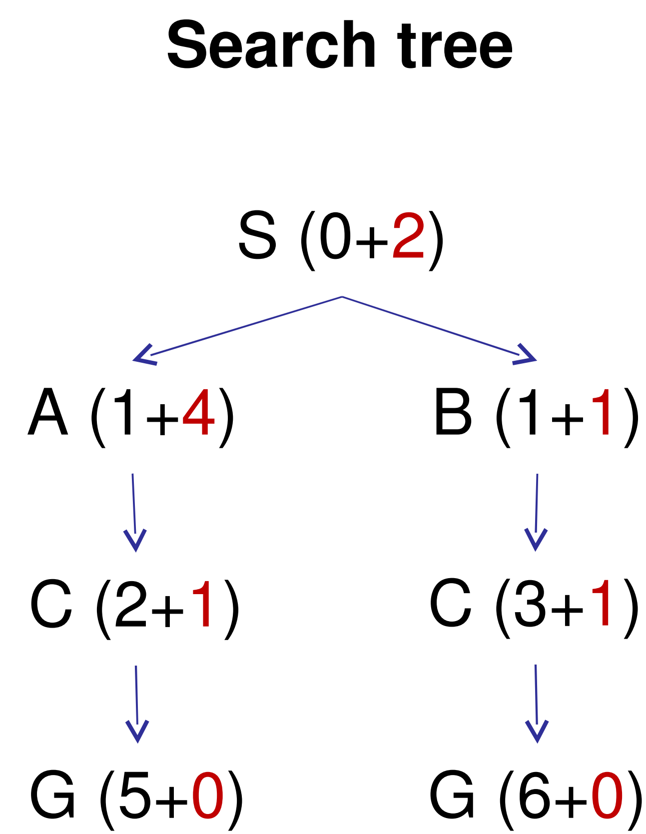
Recap: Search

- Search problem:
  - States (configurations of the world)
  - Transition function: a function from states and actions to lists of (state, cost) pairs; drawn as a graph
  - Start state and goal test
- Search tree:
  - Nodes: represent plans for reaching states
  - Plans have costs (sum of action costs)
- Search Algorithm:
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)

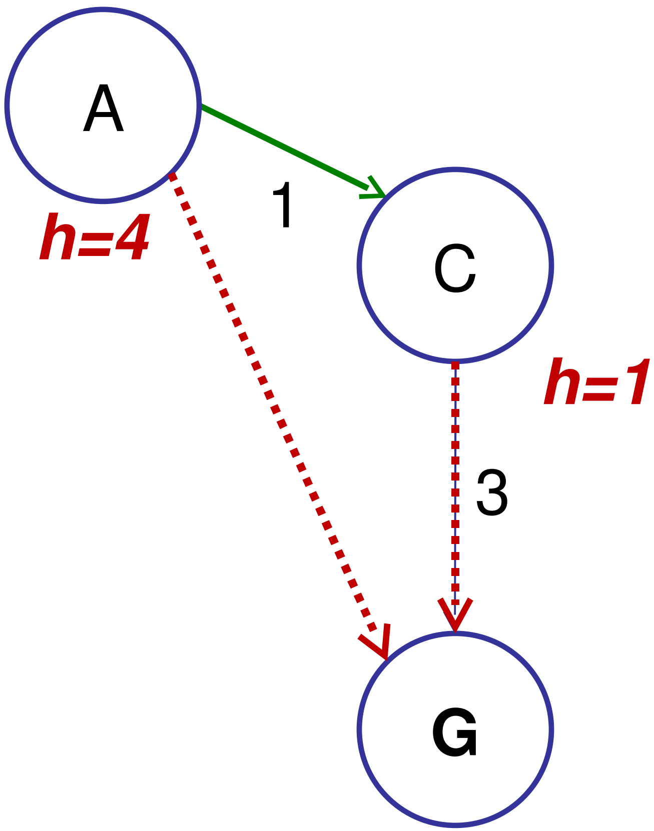
This slide deck courtesy of Dan Klein at UC Berkeley

A* Graph Search Gone Wrong?





Consistency of Heuristics



- Stronger than admissibility
- Definition:

  cost(A to C) + h(C) ≥ h(A)

  cost(A to C) ≥ h(A) - h(C)

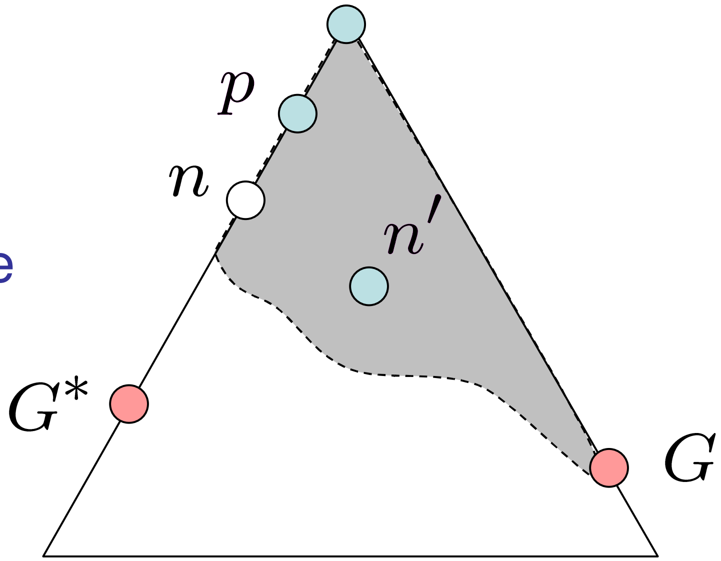
  real cost ≥ cost implied by heuristic

- Consequences:
  - The f value along a path never decreases
  - A* graph search is optimal

Optimality of A* Graph Search

Proof:

- New possible problem: some *n* on path to G* isn't in queue when we need it, because some worse *n'* for the same state dequeued and expanded first (disaster!)
- Take the highest such *n* in tree
- Let *p* be the ancestor of *n* that was on the queue when *n*' was popped
- *f(p) < f(n)* because of consistency
- *f(n) < f(n')* because *n'* is suboptimal
- *p* would have been expanded before *n*'
- Contradiction!



Optimality

- Tree search:
  - A* is optimal if heuristic is admissible (and non-negative)
  - UCS is a special case (h = 0)
- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)
- Consistency implies admissibility
- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

Summary: A*

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems

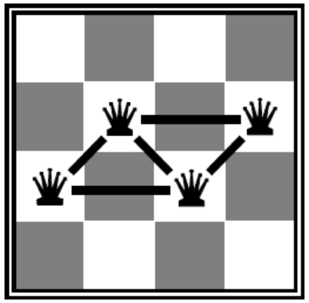
Local Search Methods

- Tree search keeps unexplored alternatives on the fringe (ensures completeness)
- Local search: improve what you have until you can't make it better
- Generally much faster and more memory efficient (but incomplete)

Types of Search Problems

- Planning problems:
  - We want a path to a solution (examples?)
  - Usually want an optimal path
  - *Incremental formulations*

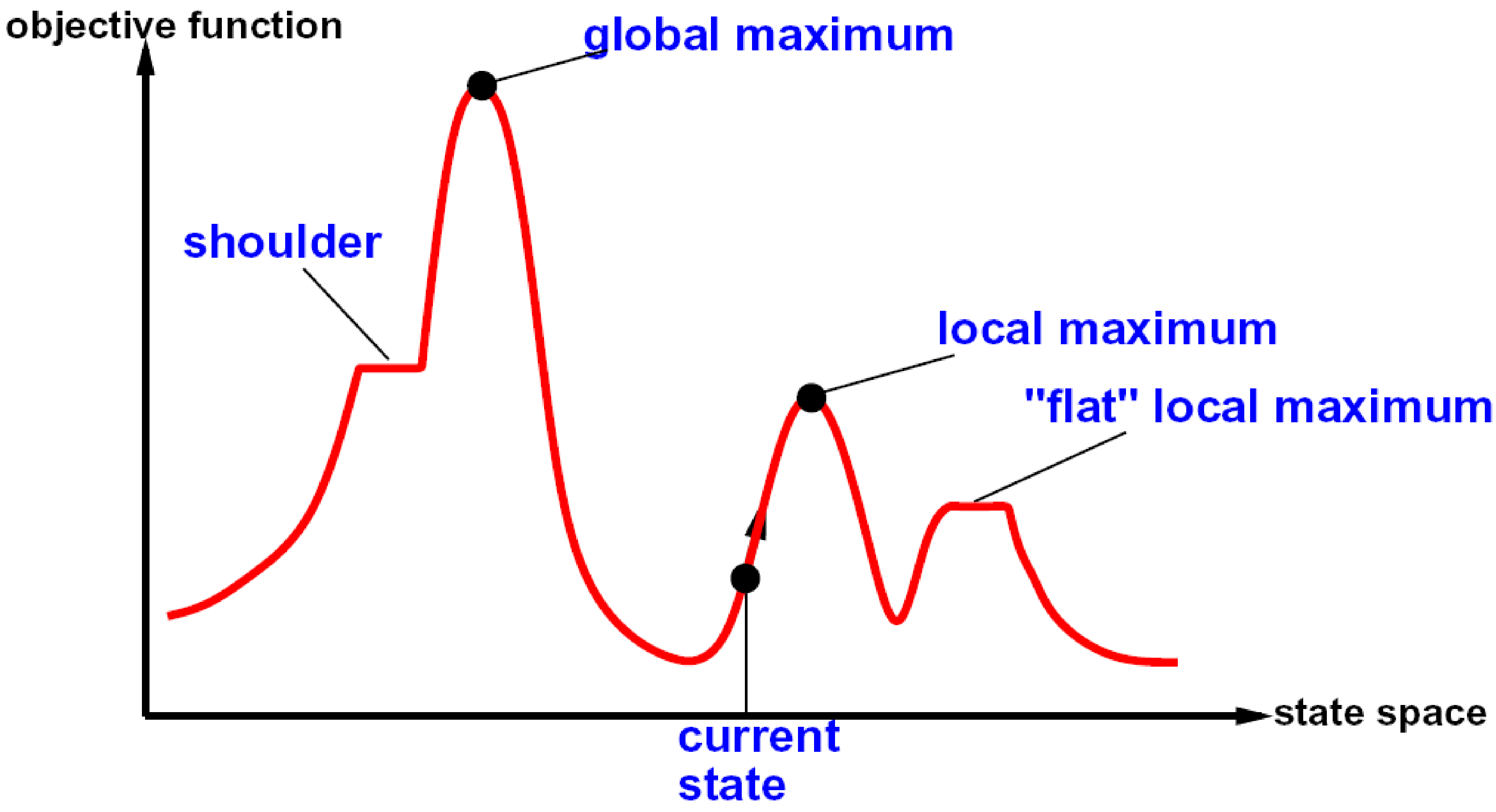
- Identification problems:
  - We actually just want to know what the goal is (examples?)
  - Usually want an optimal goal
  - *Complete-state formulations*
  - Iterative improvement algorithms



Hill Climbing

- Simple, general idea:
  - Start wherever
  - Always choose the best neighbor
  - If no neighbors have better scores than current, quit
- Why can this be a terrible idea?
  - Complete?
  - Optimal?
- What's good about it?

Hill Climbing Diagram



- Random restarts?
- Random sideways steps?

Simulated Annealing

- Idea: Escape local maxima by allowing downhill moves
  - But make them rarer as time goes on

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"
  local variables: *current*, a node
    next, a node
    T, a "temperature" controlling prob. of downward steps

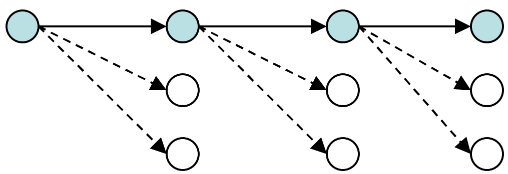
  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  for $t \leftarrow 1$ **to** ∞ **do**
    $T \leftarrow$ *schedule*[*t*]
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*]
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$

Simulated Annealing

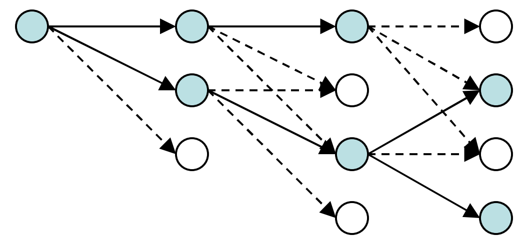
- Theoretical guarantee:
  - Stationary distribution: $p(x) \propto e^{\frac{E(x)}{kT}}$
  - If T decreased slowly enough, will converge to optimal state!
- Is this an interesting guarantee?
- Sounds like magic, but reality is reality:
  - The more downhill steps you need to escape, the less likely you are to ever make them all in a row
  - People think hard about *ridge operators* which let you jump around the space in better ways

Beam Search

- Like greedy hillclimbing search, but keep K states at all times:



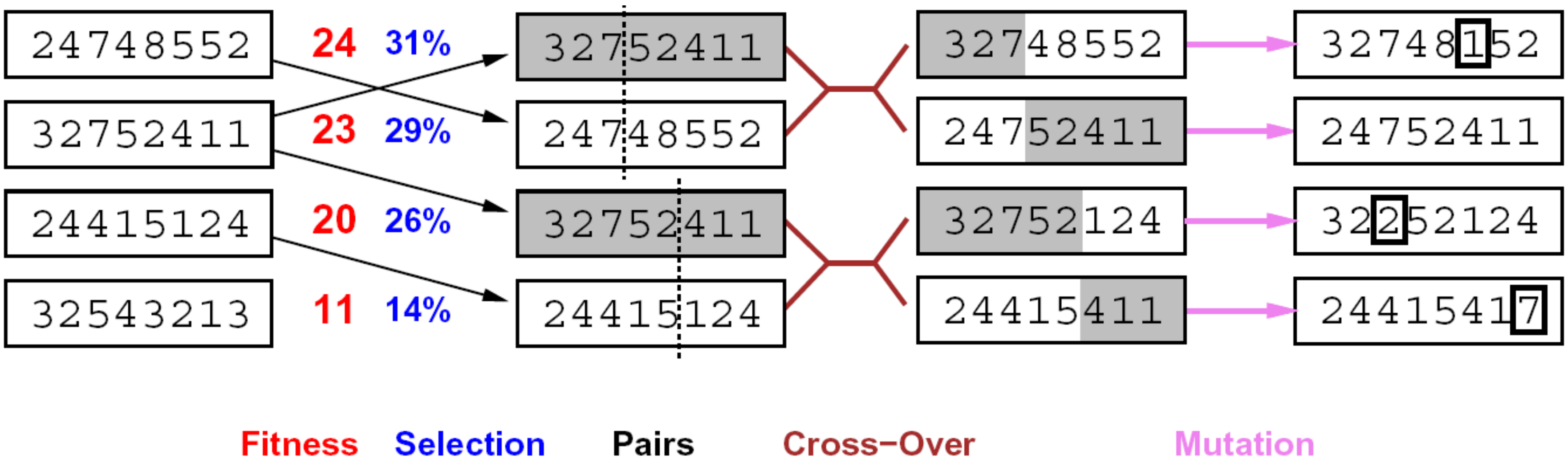
Greedy Search



Beam Search

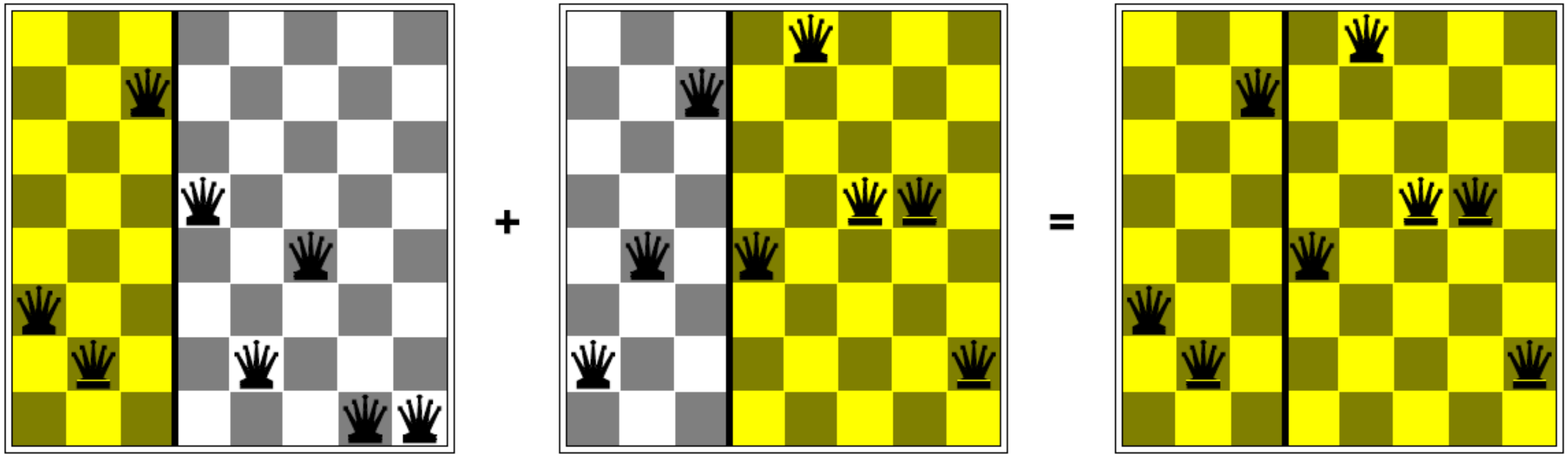
- Variables: beam size, encourage diversity?
- The best choice in MANY practical settings
- Complete? Optimal?
- Why do we still need optimal methods?

Genetic Algorithms



- Genetic algorithms use a natural selection metaphor
- Like beam search (selection), but also have pairwise crossover operators, with optional mutation
- Probably the most misunderstood, misapplied (and even maligned) technique around!

Example: N-Queens



- Why does crossover make sense here?
- When wouldn't it make sense?
- What would mutation be?
- What would a good fitness function be?