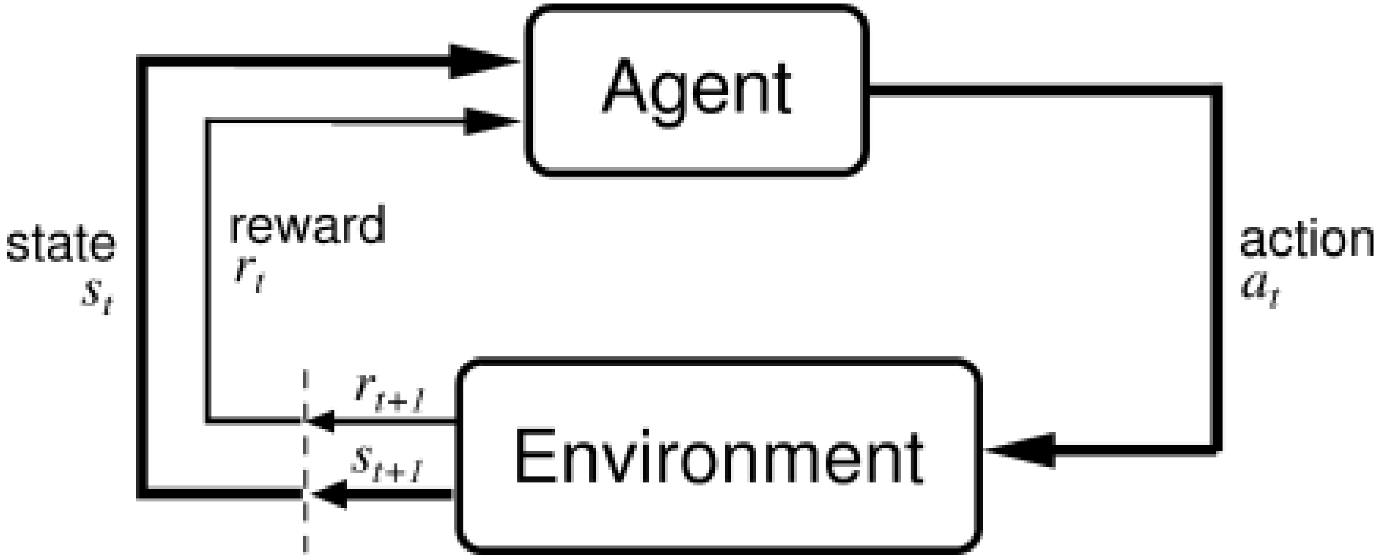
Reinforcement Learning

- Basic idea:
  - Receive feedback in the form of rewards
  - Agent's utility is defined by the reward function
  - Must (learn to) act so as to maximize expected rewards

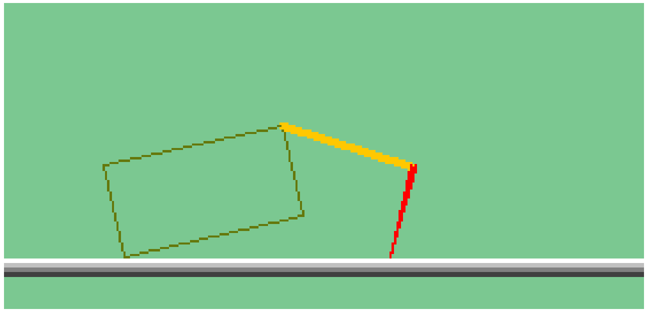


This slide deck courtesy of Dan Klein at UC Berkeley

Reinforcement Learning

- Reinforcement learning:
  - Still assume an MDP:
    - A set of states s ∈ S
    - A set of actions (per state) A
    - A model T(s,a,s')
    - A reward function R(s,a,s')
  - Still looking for a policy π (s)

  - New twist: don't know T or R
    - I.e. don't know which states are good or what the actions do
    - Must actually try actions and states out to learn



[DEMO]

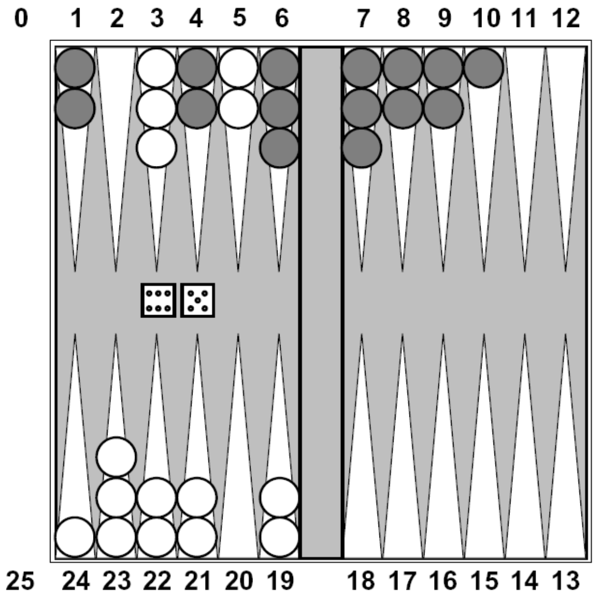
Example: Animal Learning

- RL studied experimentally for more than 60 years in psychology
  - Rewards: food, pain, hunger, drugs, etc.
  - Mechanisms and sophistication debated

- Example: foraging
  - Bees learn near-optimal foraging plan in field of artificial flowers with controlled nectar supplies
  - Bees have a direct neural connection from nectar intake measurement to motor planning area

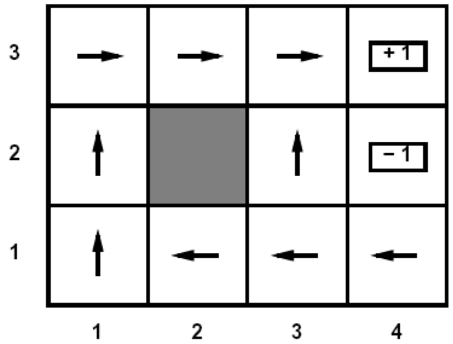
Example: Backgammon

- Reward only for win / loss in terminal states, zero otherwise
- TD-Gammon learns a function approximation to V(s) using a neural network
- Combined with depth 3 search, one of the top 3 players in the world



- You could imagine training Pacman this way…
- … but it's tricky! (It's also P3)

Passive RL



- Simplified task
  - You are given a policy π (s)
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - Goal: learn the state values
  - … what policy evaluation did

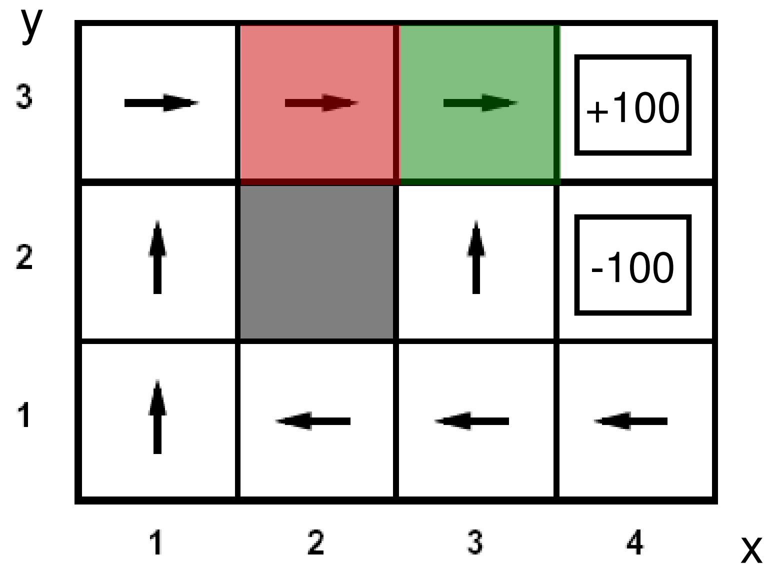
- In this case:
  - Learner "along for the ride"
  - No choice about what actions to take
  - Just execute the policy and learn from experience
  - We'll get to the active case soon
  - This is NOT offline planning! You actually take actions in the world and see what happens…

Example: Direct Evaluation

- Episodes:

(1,1) up -1
(1,2) up -1
(1,2) up -1
(1,3) right -1
(2,3) right -1
(3,3) right -1
(3,2) up -1
(3,3) right -1
(4,3) exit +100
(done)

(1,1) up -1
(1,2) up -1
(1,3) right -1
(2,3) right -1
(3,3) right -1
(3,2) up -1
(4,2) exit -100
(done)



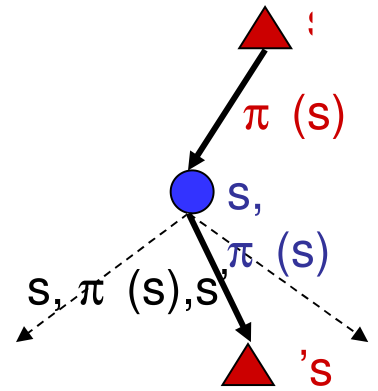
$\gamma = 1$, R = -1

V(2,3) ~ (96 + -103) / 2 = -3.5

V(3,3) ~ (99 + 97 + -102) / 3 = 31.3

Recap: Model-Based Policy Evaluation

- Simplified Bellman updates to calculate V for a fixed policy:
  - New V is expected one-step-look-ahead using current V
  - Unfortunately, need T and R



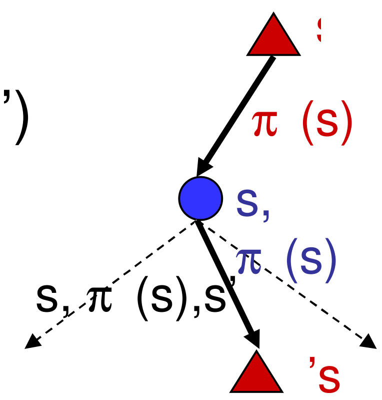
$$V_0^{\pi}(s) = 0$$

$$V_{i+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_i^{\pi}(s')]$$

Model-Based Learning

- Idea:
  - Learn the model empirically through experience
  - Solve for values as if the learned model were correct

- Simple empirical model learning
  - Count outcomes for each s,a
  - Normalize to give estimate of **T(s,a,s')**
  - Discover **R(s,a,s')** when we experience (s,a,s')



- Solving the MDP with the learned model
  - Iterative policy evaluation, for example

$$V^{\pi}_{i+1}(s) \leftarrow \sum_{s'} T(s,\pi(s),s')[R(s,\pi(s),s') + \gamma V^{\pi}_{i}(s')]$$

Example: Model-Based Learning

- Episodes:

(1,1) up -1

(1,2) up -1

(1,2) up -1

(1,3) right -1

(2,3) right -1

(3,3) right -1

(3,2) up -1

(3,3) right -1

(4,3) exit +100

(done)

(1,1) up -1

(1,2) up -1

(1,3) right -1

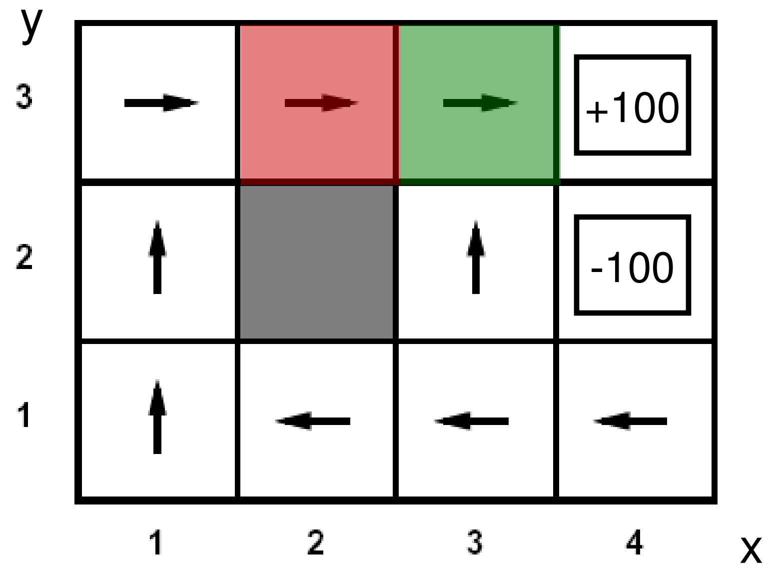
(2,3) right -1

(3,3) right -1

(3,2) up -1

(4,2) exit -100

(done)



$\gamma = 1$

T(<3,3>, right, <4,3>) = 1 / 3

T(<2,3>, right, <3,3>) = 2 / 2

Example: Expected Age

Goal: Compute expected age of cs343 students

Known P(A)

$$E[A] = \sum_a P(a) \cdot a \qquad = 0.35 \times 20 + \ldots$$

Without P(A), instead collect samples [a_1, a_2, … a_N]

Unknown P(A): "Model Based"

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a) \cdot a$$

Unknown P(A): "Model Free"

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

Model-Free Learning

- Want to compute an expectation weighted by P(x):

$$E[f(x)] = \sum_x P(x) f(x)$$

- Model-based: estimate P(x) from samples, compute expectation

$$x_i \sim P(x)$$

$$\hat{P}(x) = \text{num}(x)/N$$

$$E[f(x)] \approx \sum_x \hat{P}(x) f(x)$$

- Model-free: estimate expectation directly from samples

$$x_i \sim P(x)$$

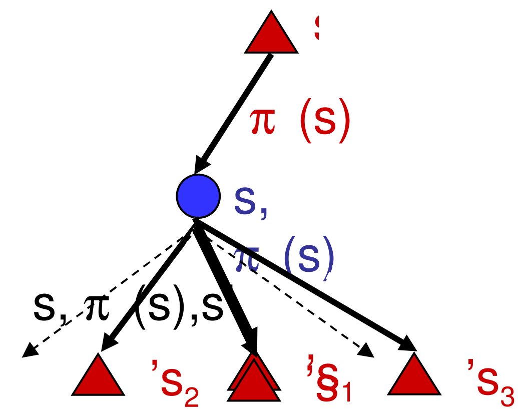
$$E[f(x)] \approx \frac{1}{N} \sum_i f(x_i)$$

- Why does this work? Because samples appear with the right frequencies!

Sample-Based Policy Evaluation?

$$V_{i+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_i^{\pi}(s')]$$

- Who needs T and R? Approximate the expectation with samples of s' (drawn from T!)



$$sample_1 = R(s, \pi(s), s_1') + \gamma V_i^{\pi}(s_1')$$

$$sample_2 = R(s, \pi(s), s_2') + \gamma V_i^{\pi}(s_2')$$

$$\ldots$$

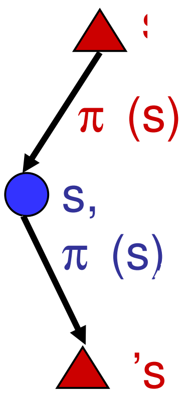
$$sample_k = R(s, \pi(s), s_k') + \gamma V_i^{\pi}(s_k')$$

$$V_{i+1}^{\pi}(s) \leftarrow \frac{1}{k} \sum_i sample_i$$

Almost! But we can't rewind time to get sample after sample from state s.

Temporal-Difference Learning

- Big idea: learn from every experience!
  - Update V(s) each time we experience (s,a,s’,r)
  - Likely s’ will contribute updates more often



- Temporal difference learning
  - Policy still fixed!
  - Move values toward value of whatever successor occurs: running average!

Sample of V(s): $$sample = R(s, \pi(s), s') + \gamma V^{\pi}(s')$$

Update to V(s): $$V^{\pi}(s) \leftarrow (1 - \alpha)V^{\pi}(s) + (\alpha)sample$$

Same update: $$V^{\pi}(s) \leftarrow V^{\pi}(s) + \alpha(sample - V^{\pi}(s))$$

Exponential Moving Average

- Exponential moving average
  - The running interpolation update

$$\bar{x}_n = (1-\alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$$

  - Makes recent samples more important

$$\bar{x}_n = \frac{x_n + (1-\alpha) \cdot x_{n-1} + (1-\alpha)^2 \cdot x_{n-2} + \ldots}{1 + (1-\alpha) + (1-\alpha)^2 + \ldots}$$

  - Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate can give converging averages

Example: TD Policy Evaluation

$$V^{\pi}(s) \leftarrow (1-\alpha)V^{\pi}(s) + \alpha\left[R(s,\pi(s),s') + \gamma V^{\pi}(s')\right]$$

(1,1) up -1

(1,2) up -1

(1,2) up -1

(1,3) right -1

(2,3) right -1

(3,3) right -1

(3,2) up -1

(3,3) right -1

(4,3) exit +100

(done)

(1,1) up -1

(1,2) up -1

(1,3) right -1

(2,3) right -1

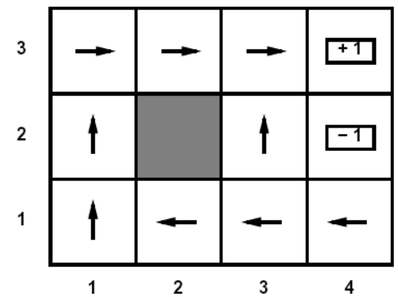
(3,3) right -1

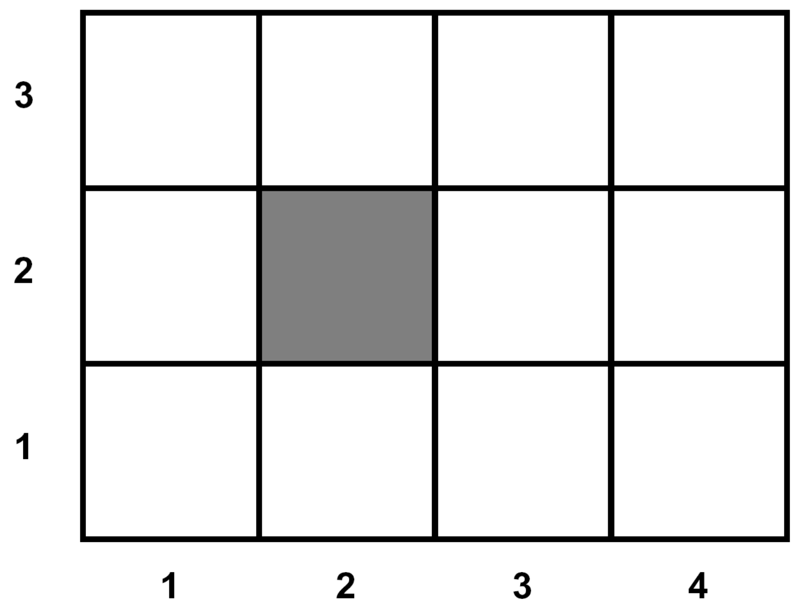
(3,2) up -1

(4,2) exit -100

(done)

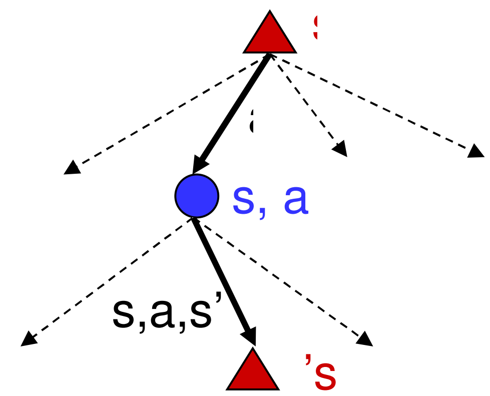
Take $\gamma = 1$, $\alpha = 0.5$





Problems with TD Value Learning

- TD value leaning is a model-free way to do policy evaluation
- However, if we want to turn values into a (new) policy, we’re sunk:

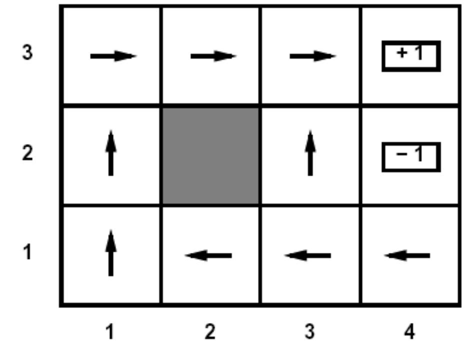


$$\pi(s) = \arg\max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s') \right]$$

- Idea: learn Q-values directly
- Makes action selection model-free too!

Active RL



- Full reinforcement learning
  - You don't know the transitions T(s,a,s')
  - You don't know the rewards R(s,a,s')
  - You can choose any actions you like
  - Goal: learn the optimal policy / values
  - … what value iteration did!

- In this case:
  - Learner makes choices!
  - Fundamental tradeoff: exploration vs. exploitation
  - This is NOT offline planning! You actually take actions in the world and find out what happens…

Detour: Q-Value Iteration

- Value iteration: find successive approx optimal values
  - Start with $V_0^*(s) = 0$, which we know is right (why?)
  - Given V_i^*, calculate the values for all states for depth i+1:

$$V_{i+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_i(s') \right]$$

- But Q-values are more useful!
  - Start with $Q_0^*(s,a) = 0$, which we know is right (why?)
  - Given Q_i^*, calculate the q-values for all q-states for depth i+1:

$$Q_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_i(s',a') \right]$$

[DEMO – Grid Q's]

Q-Learning

- Q-Learning: sample-based Q-value iteration
- Learn Q*(s,a) values
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a') \right]$$

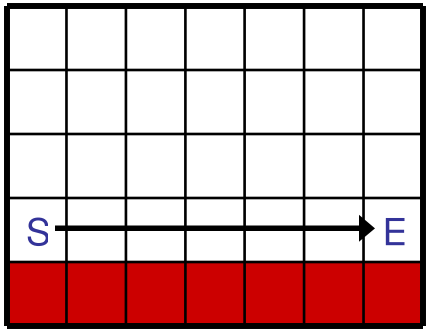
$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

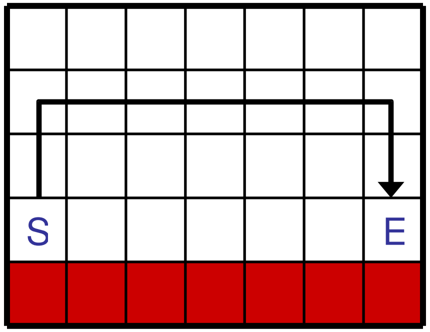
  - Incorporate the new estimate into a running average:

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[sample\right]$$

Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy
  - If you explore enough
  - If you make the learning rate small enough
  - … but not decrease it too quickly!
  - Basically doesn't matter how you select actions (!)
- Neat property: off-policy learning
  - learn optimal policy without following it (some caveats)



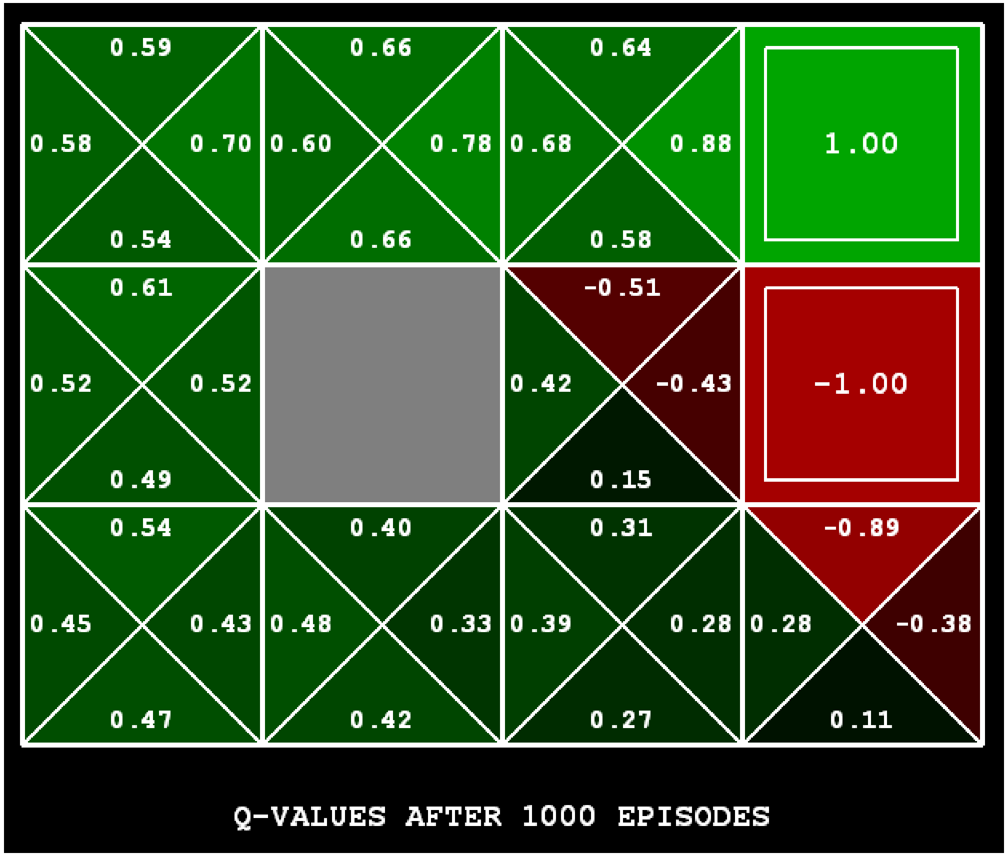


Exploration / Exploitation

- Several schemes for forcing exploration
  - Simplest: random actions (ε greedy)
    - Every time step, flip a coin
    - With probability ε, act randomly
    - With probability 1-ε, act according to current policy
  - Problems with random actions?
    - You do explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
    - Another solution: exploration functions

Q-Learning

- Q-learning produces tables of q-values:



Exploration Functions

- When to explore
  - Random actions: explore a fixed amount
  - Better idea: explore areas whose badness is not (yet) established

- Exploration function
  - Takes a value estimate and a count, and returns an optimistic utility, e.g. $f(u, n) = u + k/n$ (exact form not important)

$$Q_{i+1}(s, a) \leftarrow_\alpha R(s, a, s') + \gamma \max_{a'} Q_i(s', a')$$

$$Q_{i+1}(s, a) \leftarrow_\alpha R(s, a, s') + \gamma \max_{a'} f(Q_i(s', a'), N(s', a'))$$

The Story So Far: MDPs and RL

Things we know how to do:

- If we know the MDP
  - Compute V*, Q*, π * exactly
  - Evaluate a fixed policy π
- If we don't know the MDP
  - We can estimate the MDP then solve
  - We can estimate V for a fixed policy π
  - We can estimate Q*(s,a) for the optimal policy while executing an exploration policy

Techniques:

- Model-based DPs
  - Value Iteration
  - Policy evaluation
- Model-based RL
- Model-free RL
  - Value learning
  - Q-learning