The Story So Far: MDPs and RL

Things we know how to do:

- If we know the MDP
  - Compute V*, Q*, π * exactly
  - Evaluate a fixed policy π
- If we don't know the MDP
  - We can estimate the MDP then solve
  - We can estimate V for a fixed policy π
  - We can estimate Q*(s,a) for the optimal policy while executing an exploration policy

Techniques:

- Model-based DPs
  - Value Iteration
  - Policy evaluation
- Model-based RL
- Model-free RL
  - Value learning
  - Q-learning

This slide deck courtesy of Dan Klein at UC Berkeley

[DEMO – Grid Q's]

Q-Learning

- Q-Learning: sample-based Q-value iteration
- Learn Q*(s,a) values
  - Receive a sample (s,a,s',r)
  - Consider your old estimate: $Q(s,a)$
  - Consider your new sample estimate:

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q^*(s',a') \right]$$

$$sample = R(s,a,s') + \gamma \max_{a'} Q(s',a')$$

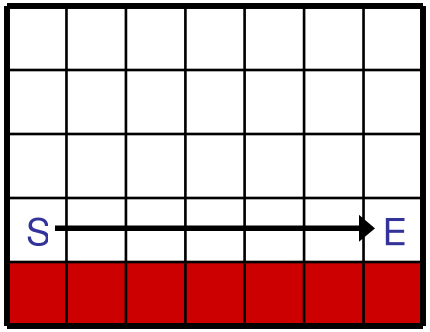
  - Incorporate the new estimate into a running average:

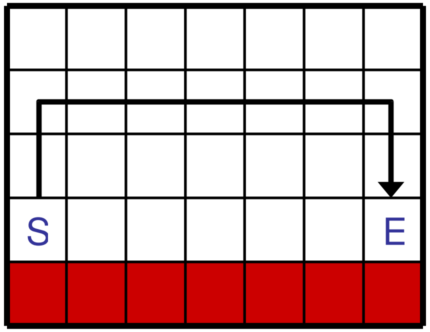
$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + (\alpha)\left[sample\right]$$

Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy
  - If you explore enough
  - If you make the learning rate small enough
  - … but not decrease it too quickly!
  - Basically doesn't matter how you select actions (!)

- Neat property: off-policy learning
  - learn optimal policy without following it (some caveats)



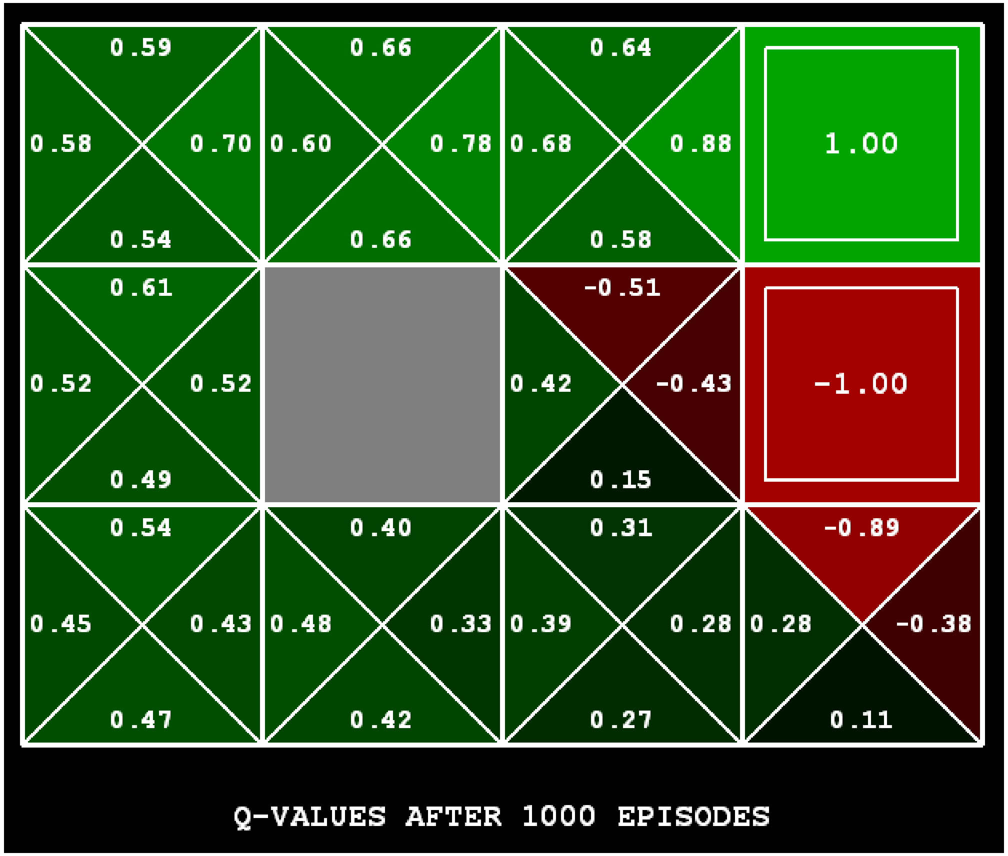


Exploration / Exploitation

- Several schemes for forcing exploration
  - Simplest: random actions (ε greedy)
    - Every time step, flip a coin
    - With probability ε, act randomly
    - With probability 1-ε, act according to current policy
  - Problems with random actions?
    - You do explore the space, but keep thrashing around once learning is done
    - One solution: lower ε over time
    - Another solution: exploration functions

Q-Learning

- Q-learning produces tables of q-values:



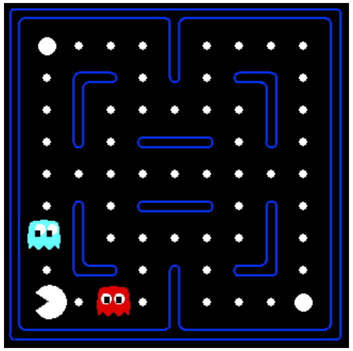
Q-Learning

- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory

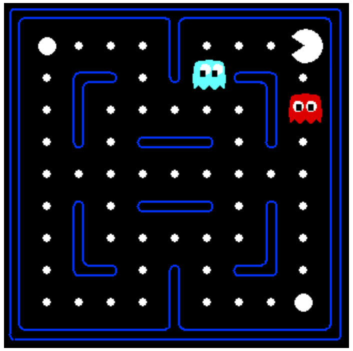
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar states
  - This is a fundamental idea in machine learning, and we'll see it over and over again

Example: Pacman

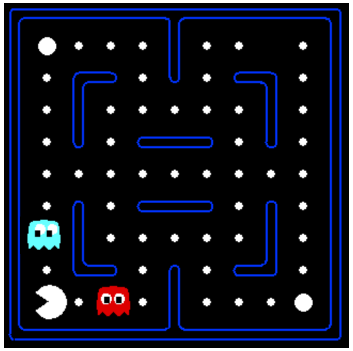
- Let's say we discover through experience that this state is bad:



- In naïve q learning, we know nothing about this state or its q states:

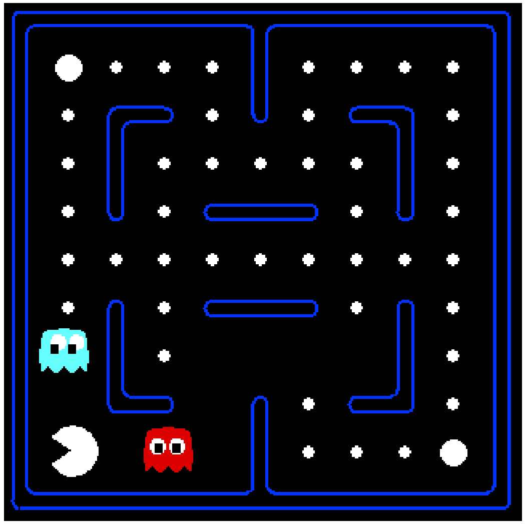


- Or even this one!



Feature-Based Representations

- Solution: describe a state using a vector of features
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)^2
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



Linear Feature Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but be very different in value!

Function Approximation

$$Q(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$$

- Q-learning with linear q-functions:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \, [error]$$

$$w_i \leftarrow w_i + \alpha \, [error] \, f_i(s,a)$$

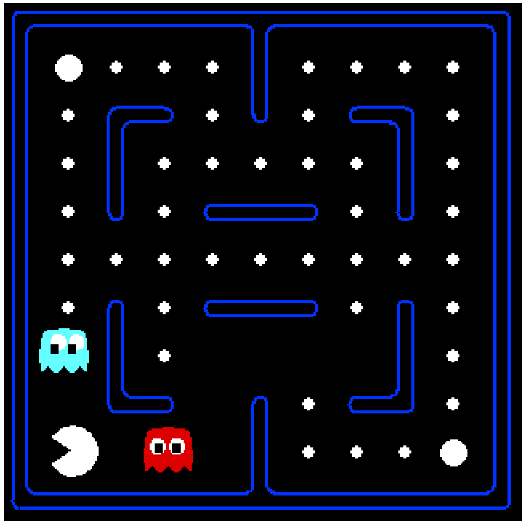
- Intuitive interpretation:
  - Adjust weights of active features
  - E.g. if something unexpectedly bad happens, disprefer all states with that state's features
- Formal justification: online least squares

Example: Q-Pacman

$$Q(s,a) = 4.0 f_{DOT}(s,a) - 1.0 f_{GST}(s,a)$$

$$f_{DOT}(s, \text{NORTH}) = 0.5$$

$$f_{GST}(s, \text{NORTH}) = 1.0$$



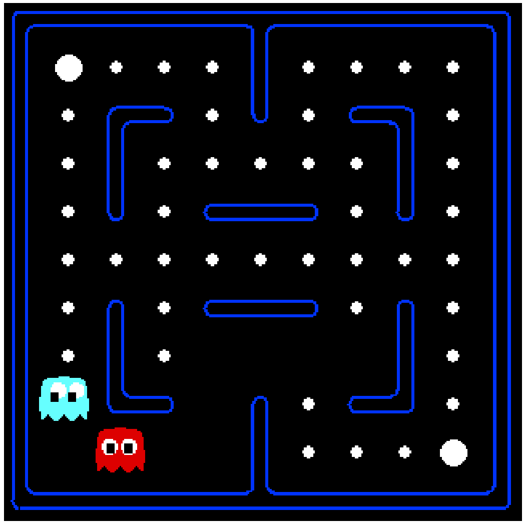
$$Q(s,a) = +1$$

$$R(s,a,s') = -500$$

$$error = -501$$

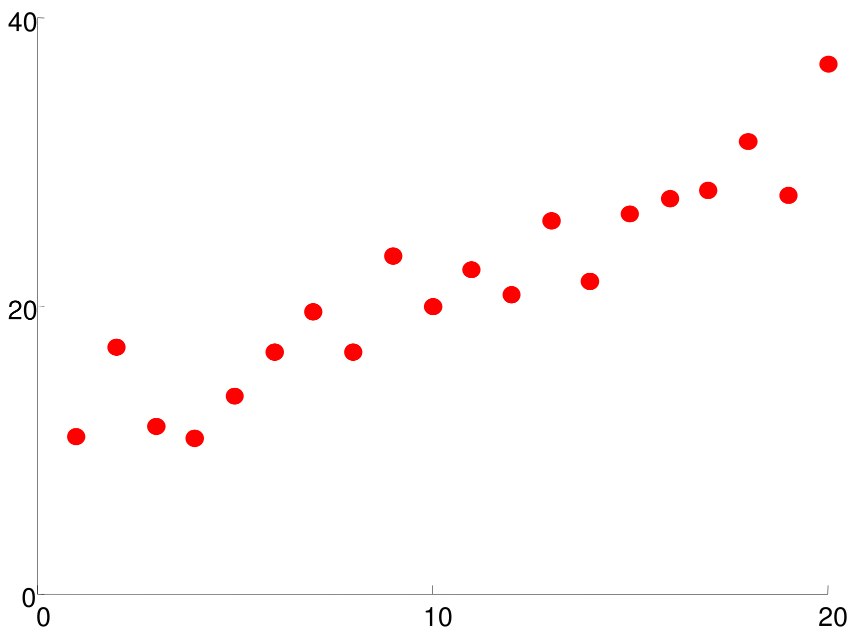
$$w_{DOT} \leftarrow 4.0 + \alpha \, [-501] \, 0.5$$

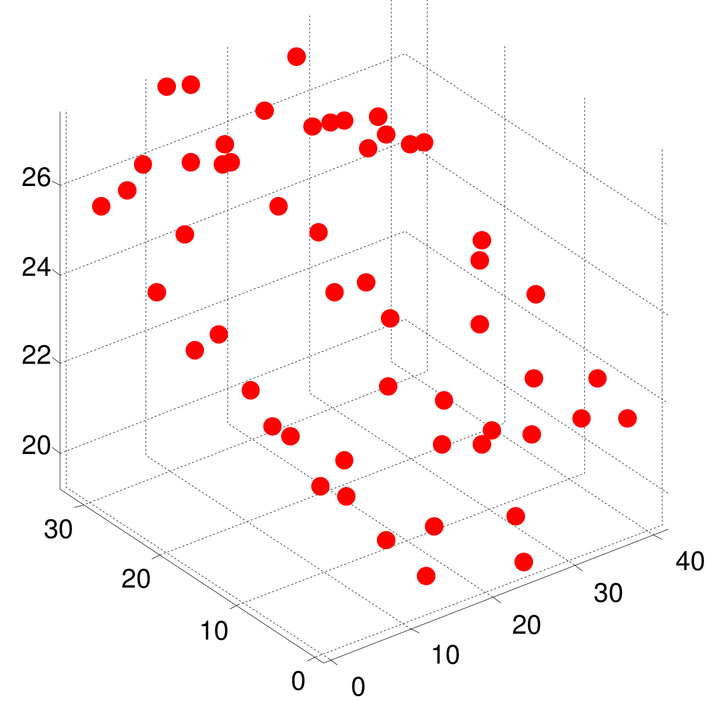
$$w_{GST} \leftarrow -1.0 + \alpha \, [-501] \, 1.0$$



$$Q(s,a) = 3.0 f_{DOT}(s,a) - 3.0 f_{GST}(s,a)$$

Linear regression

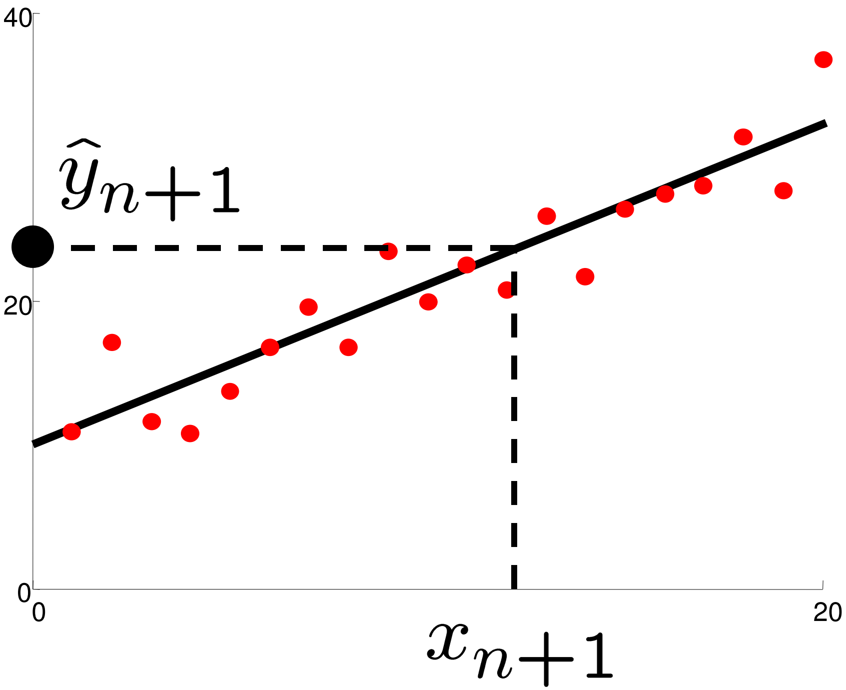


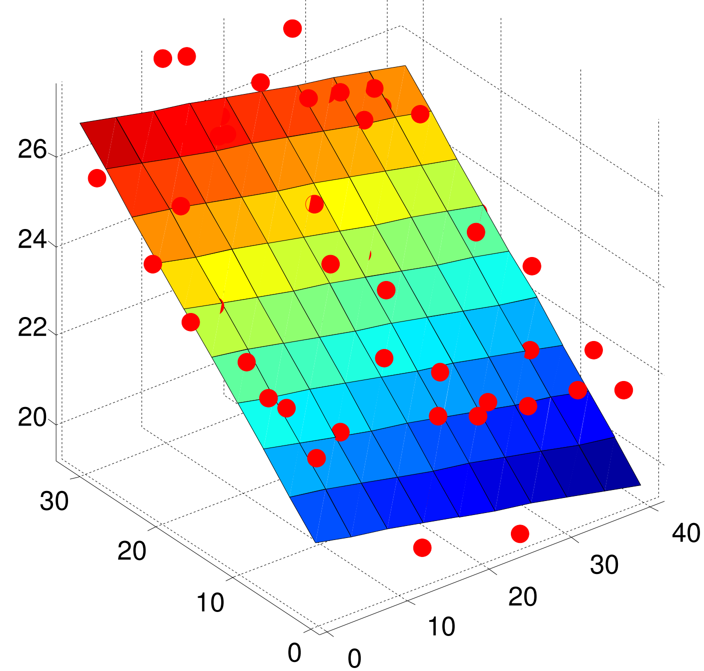


Given examples $(x_i, y_i)_{i=1\ldots n}$

Predict y_{n+1} given a new point x_{n+1}

Linear regression





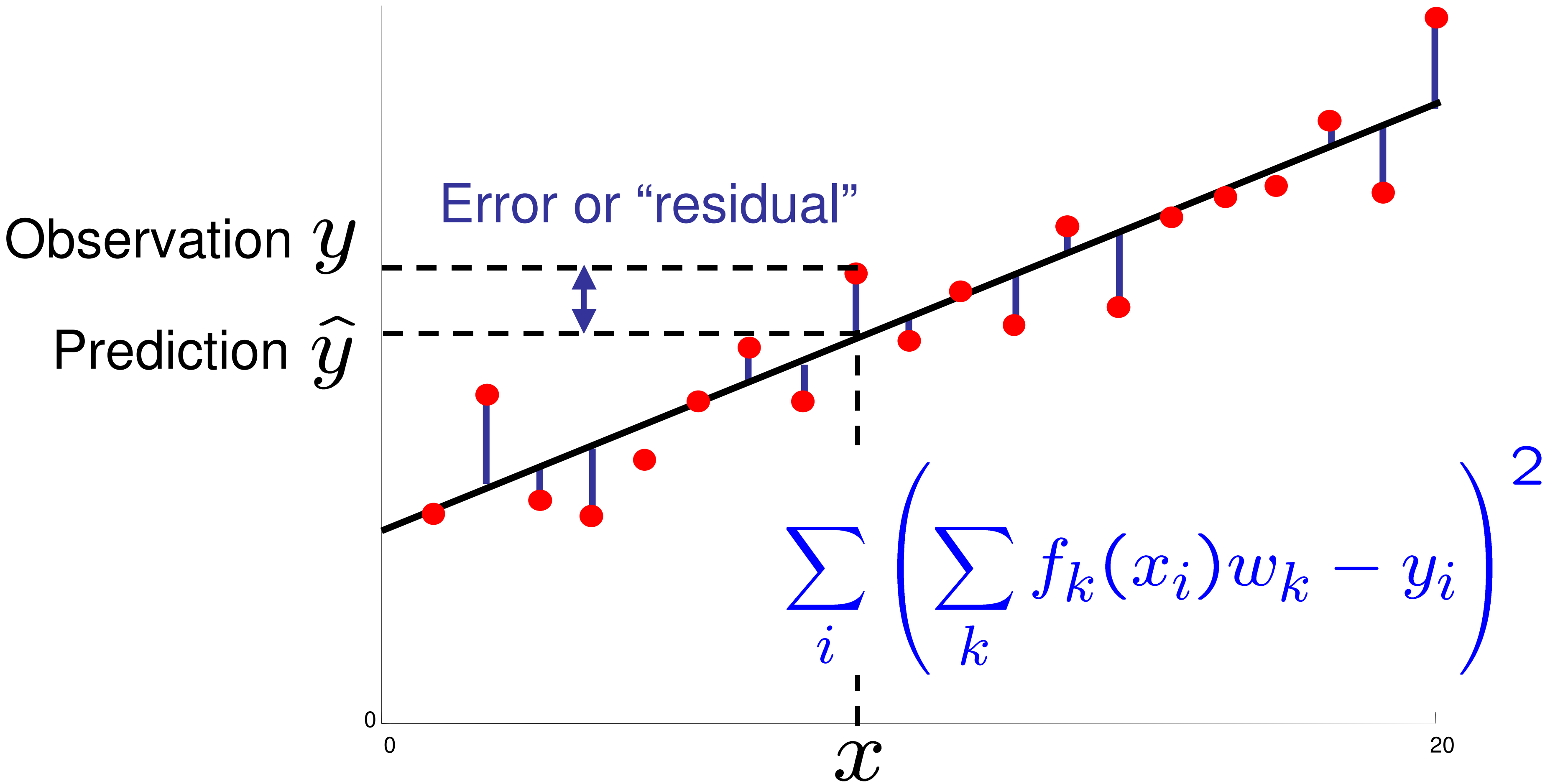
Prediction

$$\widehat{y}_i = w_0 + w_1 x_i$$

Prediction

$$\widehat{y}_i = w_0 + w_1 x_{i,1} + w_2 x_{i,2}$$

Ordinary Least Squares (OLS)



Minimizing Error

$$E(w) = \frac{1}{2}\sum_i \left(\sum_k f_k(x_i)w_k - y_i\right)^2$$

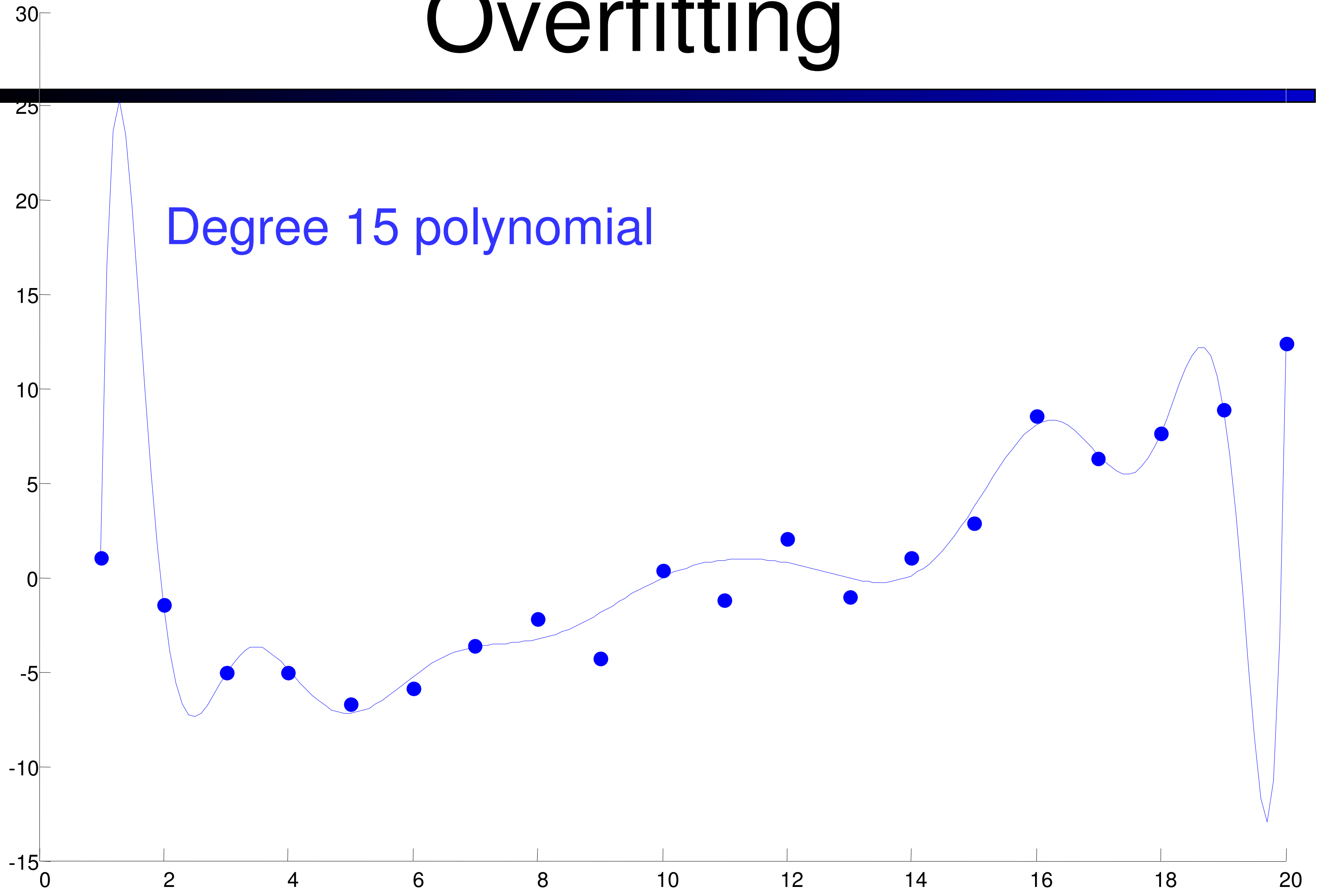
$$\frac{\partial E}{\partial w_m} = \sum_i \left(\sum_k f_k(x_i)w_k - y_i\right) f_m(x_i)$$

$$E \leftarrow E + \alpha \sum_i \left(\sum_k f_k(x_i)w_k - y_i\right) f_m(x_i)$$

Value update explained:

$$w_i \leftarrow w_i + \alpha\,[error]\,f_i(s,a)$$

Overfitting



[DEMO]

Policy Search

Policy Search

- Problem: often the feature-based policies that work well aren't the ones that approximate V / Q best
  - E.g. your value functions from project 2 were probably horrible estimates of future rewards, but they still produced good decisions
  - We'll see this distinction between modeling and prediction again later in the course

- Solution: learn the policy that maximizes rewards rather than the value that predicts rewards

- This is the idea behind policy search, such as what controlled the upside-down helicopter

Policy Search

- Simplest policy search:
  - Start with an initial linear value function or q-function
  - Nudge each feature weight up and down and see if your policy is better than before
- Problems:
  - How do we tell the policy got better?
  - Need to run many sample episodes!
  - If there are a lot of features, this can be impractical

Policy Search*

- Advanced policy search:
  - Write a stochastic (soft) policy:

$$\pi_w(s) \propto e^{\sum_i w_i f_i(s,a)}$$

  - Turns out you can efficiently approximate the derivative of the returns with respect to the parameters w (details in the book, but you don't have to know them)
  - Take uphill steps, recalculate derivatives, etc.

Take a Deep Breath…

- We’re done with search and MDPs!
- Next, we’ll look at how to reason with probabilities
  - Diagnosis
  - Tracking objects
  - … lots more!
- Last part of course: machine learning, classical planning

A (Short) History of AI

- 1940-1950: Early days
  - 1943: McCulloch & Pitts: Boolean circuit model of brain
  - 1950: Turing's “Computing Machinery and Intelligence”
- 1950—70: Excitement: Look, Ma, no hands!
  - 1950s: Early AI programs, including Samuel's checkers program, Newell & Simon's Logic Theorist, Gelernter's Geometry Engine
  - 1956: Dartmouth meeting: “Artificial Intelligence” adopted
  - 1965: Robinson's complete algorithm for logical reasoning
- 1970—88: Knowledge-based approaches
  - 1969—79: Early development of knowledge-based systems
  - 1980—88: Expert systems industry booms
  - 1988—93: Expert systems industry busts: “AI Winter”
- 1988—: Statistical approaches
  - Resurgence of probability, focus on uncertainty
  - General increase in technical depth
  - Agents and learning systems… “AI Spring”?
- 2000—: Where are we now?