CS344M
Autonomous Multiagent Systems

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The University of Texas at Austin
Good Afternoon, Colleagues

Are there any questions?
Logistics

• Progress reports due in 1 week
Logistics

- Progress reports due in 1 week
- DigCS
## Mixed strategy equilibrium

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Action 1</td>
</tr>
<tr>
<td>Action 1</td>
<td>3,7</td>
</tr>
<tr>
<td>Action 2</td>
<td>6,5</td>
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Mixed strategy equilibrium

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- What if player 2 picks action 1 3/4 of the time?
- What if player 2 picks action 1 1/4 of the time?
- Player 1 must be indifferent between actions 1 and 2
- Player 2 must be indifferent between actions 1 and 2

Do actual numbers matter?
Correlated Equilibria
Correlated Equilibria

Sometimes mixing isn’t enough: Bach/Stravinsky

<table>
<thead>
<tr>
<th></th>
<th>Wife</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S</td>
<td>B</td>
</tr>
<tr>
<td>S</td>
<td>2,1</td>
<td>0,0</td>
</tr>
<tr>
<td>Me</td>
<td>B</td>
<td>0,0</td>
</tr>
<tr>
<td></td>
<td>0,0</td>
<td>1,2</td>
</tr>
</tbody>
</table>

Want only S,S or B,B - 50% each
Focal points

- We will both be in Paris for some time in June.
- We both know that we will both be there on the 15th.
Focal points

- We will both be in Paris for some time in June.
- We both know that we will both be there on the 15th.
- Something happens so that we must meet on that day.
- We have no way of getting in touch.
Focal points

• We will both be in Paris for some time in June.

• We both know that we will both be there on the 15th.

• Something happens so that we must meet on that day.

• We have no way of getting in touch.

• When and where?
Focal points

- We will both be in Paris for some time in June.
- We both know that we will both be there on the 15th.
- Something happens so that we must meet on that day.
- We have no way of getting in touch.
- When and where?
- What are the Nash equilibria?
Incomplete Information Games

- We each get one of 3 cards: 1, 2, 3
- If we both fold, we both lose nothing
- If one raises and one folds, the raiser gets 1
- If both raise, the one with the higher card gets 5
- Zero sum
Incomplete Information Games

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<tr>
<th>Card</th>
<th>R</th>
<th>F</th>
<th>Card 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>5, -5</td>
<td>1, -1</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>-1, 1</td>
<td>0, 0</td>
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### Incomplete Information Games

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## Incomplete Information Games

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<td></td>
</tr>
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<td>1, -1</td>
</tr>
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<td></td>
<td></td>
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<td></td>
</tr>
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<td>R</td>
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<td>1, -1</td>
</tr>
<tr>
<td>Card 1</td>
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<tr>
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Bayes-Nash Equilibrium

• $3 \Rightarrow \text{raise}$
Bayes-Nash Equilibrium

- 3 ⇒ raise
- 1 ⇒ fold (no matter what the other one does with 2)
Bayes-Nash Equilibrium

- $3 \Rightarrow$ raise
- $1 \Rightarrow$ fold (no matter what the other one does with 2)
- $2 \Rightarrow$ ?
Bayes-Nash Equilibrium

- 3 ⇒ raise
- 1 ⇒ fold (no matter what the other one does with 2)
- 2 ⇒ ?
  - Raise: \((.5)(-5) + (.5)(1) = -2\)
  - Fold: \((.5)(-1) + (.5)(0) = -.5\)
Bayes-Nash Equilibrium

- 3 $\Rightarrow$ raise

- 1 $\Rightarrow$ fold (no matter what the other one does with 2)

- 2 $\Rightarrow$ ?
  - Raise: $(.5)(-5) + (.5)(1) = -2$
  - Fold: $(.5)(-1) + (.5)(0) = -.5$
  - Always fold!
Bayes-Nash Equilibrium

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  - Raise: $(.5)(-5) + (.5)(1) = -2$
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  - Always fold!
  - Bayes-Nash: both players Raise if 3, otherwise Fold
Bayes-Nash Equilibrium

- 3 ⇒ raise
- 1 ⇒ fold (no matter what the other one does with 2)
- 2 ⇒ ?
  - Raise: \((.5)(-5) + (.5)(1) = -2\)
  - Fold: \((.5)(-1) + (.5)(0) = -.5\)
  - Always fold!
- Bayes-Nash: both players Raise if 3, otherwise Fold

With more numbers and/or different payoffs, bluffing can be a part of the Nash Equilibrium
Discussion

• How useful is the concept of Nash equilibrium?
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• What if one player isn’t rational?
Discussion

- How useful is the concept of Nash equilibrium?
- What if one player isn’t rational?
- What can’t game theory simulate?
Repeated games

- Book slides
Repeated games

- Book slides
- Tournaments on resources page
Repeated games

- Book slides
- Tournaments on resources page
- Threats slides
Repeated games

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- Threats slides
- Doran’s ICML slides