Good Afternoon, Colleagues

Are there any questions?
Good Afternoon, Colleagues

Are there any questions?

- Mixed Nash equilibria?
- What can’t game theory simulate?
- What if one player isn’t rational?
- Doran’s research
Logistics

- Project progress reports due next week
Logistics

- Project progress reports due next week
- Thoughts on faculty candidate?
Class Discussion

Matt Wilson on a multiagent game
Bach/Stravinsky

- My wife and I agree to meet at a concert
Bach/Stravinsky

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- Unfortunately, there are 2: Bach and Stravinsky
My wife and I agree to meet at a concert.

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No time to get in touch with each other.
Bach/Stravinsky

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- I prefer Stravinsky, she prefers Bach
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- But most of all, we want to be together
Bach/Stravinsky

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- Unfortunately, there are 2: Bach and Stravinsky
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- I prefer Stravinsky, she prefers Bach
- But most of all, we want to be together
- Propose a payoff matrix
<table>
<thead>
<tr>
<th></th>
<th>Wife</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S</td>
<td>B</td>
</tr>
<tr>
<td>S</td>
<td>2,1</td>
<td>0,0</td>
</tr>
<tr>
<td>Me</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0,0</td>
<td>1,2</td>
</tr>
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Correlated Equilibria

Sometimes mixing isn’t enough: Bach/Stravinsky

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Peter Stone
## Correlated Equilibria

Sometimes mixing isn’t enough: Bach/Stravinsky

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Want only S,S or B,B - 50% each
Focal points

- We will both be in Paris for some time in June.
- We both know that we will both be there on the 15th.
Focal points

- We will both be in Paris for some time in June.
- We both know that we will both be there on the 15th.
- Something happens so that we must meet on that day.
- We have no way of getting in touch.
Focal points

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• When and where?
Focal points

• We will both be in Paris for some time in June.
• We both know that we will both be there on the 15th.
• Something happens so that we must meet on that day.
• We have no way of getting in touch.
• When and where?
• What are the Nash equilibria?
Incomplete Information Games

- We each get one of 3 cards: 1, 2, 3
- If we both fold, we both lose nothing
- If one raises and one folds, the raiser gets 1
- If both raise, the one with the higher card gets 5
- Zero sum
Incomplete Information Games

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<thead>
<tr>
<th>Card ?</th>
<th>R</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>5, -5</td>
<td>1, -1</td>
</tr>
<tr>
<td>F</td>
<td>-1, 1</td>
<td>0, 0</td>
</tr>
</tbody>
</table>
Incomplete Information Games

Card 3

<table>
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<tr>
<th></th>
<th>R</th>
<th>F</th>
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<tbody>
<tr>
<td>R</td>
<td>5,-5</td>
<td>1,-1</td>
</tr>
<tr>
<td>F</td>
<td>-1,1</td>
<td>0,0</td>
</tr>
</tbody>
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# Incomplete Information Games

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<tr>
<td>R 5,-5</td>
<td>1,-1</td>
<td></td>
</tr>
<tr>
<td>F -1,1</td>
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<tbody>
<tr>
<td>R -5,5</td>
<td>1,-1</td>
<td></td>
</tr>
<tr>
<td>F -1,1</td>
<td>0,0</td>
<td></td>
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Bayes-Nash Equilibrium

- 3 ⇒ raise
Bayes-Nash Equilibrium

- 3 ⇒ raise
- 1 ⇒ fold (no matter what the other one does with 2)
Bayes-Nash Equilibrium

- 3 ⇒ raise
- 1 ⇒ fold (no matter what the other one does with 2)
- 2 ⇒ ?
Bayes-Nash Equilibrium

- 3 ⇒ raise
- 1 ⇒ fold (no matter what the other one does with 2)
- 2 ⇒ ?
  - Raise: \((.5)(-5) + (.5)(1) = -2\)
  - Fold: \((.5)(-1) + (.5)(0) = -.5\)
Bayes-Nash Equilibrium

- $3 \Rightarrow$ raise
- $1 \Rightarrow$ fold (no matter what the other one does with 2)
- $2 \Rightarrow$ ?
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  - Always fold!
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  - Always fold!
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With more numbers and/or different payoffs, bluffing can be a part of the Nash Equilibrium
## Stackelburg Game

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<tr>
<th>Player 1</th>
<th>Action 1</th>
<th>Action 2</th>
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<tbody>
<tr>
<td>Action 1</td>
<td>1,0</td>
<td>3,2</td>
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<td>2,1</td>
<td>4,0</td>
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- Nash equilibrium?
- Action 2 is dominant for Player 1. End of story?
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  Threats slides
Discussion

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– Need to do well against some set of agents, never too poorly, and well against yourself.
Stochastic Games

- Tutorial slides