CS344M
Autonomous Multiagent Systems
Spring 2008

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Good Afternoon, Colleagues

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- Mixed Nash equilibria?
- What can't game theory simulate?
- What if one player isn't rational?
- Doran's research

Logistics

- Project progress reports due next week

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- Thoughts on faculty candidate?

Class Discussion

Matt Wilson on a multiagent game

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- But most of all, we want to be together
- Propose a payoff matrix

Bach/Stravinsky

		Wife	
		S	B
Me	S	2,1	0,0
	B	0,0	1,2

Correlated Equilibria

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Want only S,S or B,B - 50% each

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- When and where?
- What are the Nash equilibria?

Incomplete Information Games

- We each get one of 3 cards: 1,2,3
- If we both fold, we both lose nothing
- If one raises and one folds, the raiser gets 1
- If both raise, the one with the higher card gets 5
- Zero sum

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		Card ?	
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Card 3	R	5,-5	1,-1
	F	-1,1	0,0

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> With more numbers and/or different payoffs, bluffing can be a part of the Nash Equilibrium

Stackelburg Game

		Player 2	
		Action 1	Action 2
Player 1	Action 1	1,0	3,2
	Action 2	2,1	4,0

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Threats slides

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– Need to do well against some set of agents, never too poorly, and well against yourself.

Stochastic Games

- Tutorial slides