Good Afternoon, Colleagues

Are there any questions?
Good Afternoon, Colleagues

Are there any questions?

- What if you don’t know outcomes ahead of time?
- Can strategies in the iterated case be automated?
Logistics

- Project progress reports due next week
T-test vs. Paired T-test

- Test: Your team better than UvA vs. CMUnited
T-test vs. Paired T-test

- Test: Your team better than UvA vs. CMUnited
- Test: Your team better than UvA vs. a set of 20 opponents
T-test vs. Paired T-test

- Test: Your team better than UvA vs. CMUnited
- Test: Your team better than UvA vs. a set of 20 opponents
- What if neither is significant?
Student-led discussion

• Zac on real-world uses of game theory
## Mixed strategy equilibrium

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Action 1</td>
<td>2,2</td>
</tr>
<tr>
<td>Action 2</td>
<td>3,1</td>
</tr>
</tbody>
</table>
Mixed strategy equilibrium

<table>
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<tr>
<th>Player 1</th>
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<tbody>
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<td>Action 1</td>
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<td>3,1</td>
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<table>
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<th>Player 2</th>
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</tr>
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<tr>
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- Pure strategy Nash equilibrium?
Mixed strategy equilibrium

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- Pure strategy Nash equilibrium?
- Mixed strategy Nash equilibrium?
Correlated Equilibria

Sometimes mixing isn’t enough: Bach/Stravinsky

<table>
<thead>
<tr>
<th></th>
<th>Wife</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S</td>
</tr>
<tr>
<td>S</td>
<td>2,1</td>
</tr>
<tr>
<td>Me</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>0,0</td>
</tr>
</tbody>
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Sometimes mixing isn’t enough: Bach/Stravinsky

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<td>B 0,0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Me</td>
<td>B 0,0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>1,2</td>
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</tbody>
</table>

Want only S,S or B,B - 50% each
Focal points

• We will both be in Paris for some time in June.

• We both know that we will both be there on the 15th.
Focal points

• We will both be in Paris for some time in June.

• We both know that we will both be there on the 15th.

• Something happens so that we must meet on that day

• We have no way of getting in touch.
Focal points

- We will both be in Paris for some time in June.
- We both know that we will both be there on the 15th.
- Something happens so that we must meet on that day.
- We have no way of getting in touch.
- When and where?
Focal points

• We will both be in Paris for some time in June.
• We both know that we will both be there on the 15th.
• Something happens so that we must meet on that day
• We have no way of getting in touch.
• When and where?
• What are the Nash equilibria?
Incomplete Information Games

- We each get one of 3 cards: 1, 2, 3
- If we both fold, we both lose nothing
- If one raises and one folds, the raiser gets 1
- If both raise, the one with the higher card gets 5
- Zero sum
**Incomplete Information Games**

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<tr>
<th>Card</th>
<th>R</th>
<th>F</th>
</tr>
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<tbody>
<tr>
<td>R</td>
<td>5, -5</td>
<td>1, -1</td>
</tr>
<tr>
<td>F</td>
<td>-1, 1</td>
<td>0, 0</td>
</tr>
</tbody>
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# Incomplete Information Games

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<td>1, -1</td>
</tr>
<tr>
<td>Card 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>-1, 1</td>
<td>0, 0</td>
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# Incomplete Information Games

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<tr>
<th>Card</th>
<th>R -5,5</th>
<th>1,-1</th>
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<tr>
<td>Card 1</td>
<td>F -1,1</td>
<td>0,0</td>
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Bayes-Nash Equilibrium

- $3 \Rightarrow \text{raise}$
Bayes-Nash Equilibrium

- 3 $\Rightarrow$ raise

- 1 $\Rightarrow$ fold (no matter what the other one does with 2)
Bayes-Nash Equilibrium

- 3 ⇒ raise

- 1 ⇒ fold (no matter what the other one does with 2)

- 2 ⇒ ?
Bayes-Nash Equilibrium

• 3 ⇒ raise

• 1 ⇒ fold (no matter what the other one does with 2)

• 2 ⇒ 
  – Raise: (.5)(-5) + (.5)(1) = -2
  – Fold: (.5)(-1) + (.5)(0) = -.5
Bayes-Nash Equilibrium

- 3 ⇒ raise
- 1 ⇒ fold (no matter what the other one does with 2)
- 2 ⇒ ?
  - Raise: \(.5\)(-5) + (.5)(1) = -2
  - Fold: (.5)(-1) + (.5)(0) = -.5
  - Always fold!
Bayes-Nash Equilibrium

- 3 ⇒ raise
- 1 ⇒ fold (no matter what the other one does with 2)
- 2 ⇒ ?
  - Raise: \((0.5)(-5) + (0.5)(1) = -2\)
  - Fold: \((0.5)(-1) + (0.5)(0) = -0.5\)
  - Always fold!
  - Bayes-Nash: both players Raise if 3, otherwise Fold
Bayes-Nash Equilibrium

- 3 ⇒ raise
- 1 ⇒ fold (no matter what the other one does with 2)
- 2 ⇒ ?
  - Raise: (.5)(-5) + (.5)(1) = -2
  - Fold: (.5)(-1) + (.5)(0) = -.5
  - Always fold!
  - Bayes-Nash: both players Raise if 3, otherwise Fold

With more numbers and/or different payoffs, bluffing can be a part of the Nash Equilibrium