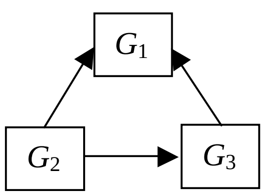
The following is an attempt to describe in my own words the variable elimination algorithm for determining optimal joint actions of coordinating agents. Each agent is eliminated one at a time from a coordination graph by deciding the optimal local payoff based on the payoff functions of dependent agents, then notifying the other agents of this conditional decision. Specifically, the procedure for eliminating agent i is:

1. for each agent j in the graph that has a payoff rule dependent on the action of agent i (child nodes of agent i)
    - remove the dependent payoff rule of agent j
    - add it to the payoff function for agent i
2. maximize the local payoff by determining which combinations of agent actions will produce the greatest payoffs given the payoff function for agent i
3. distribute this conditional strategy to the agent that will be eliminated next
4. eliminate agent i from the graph
5. update the coordinated graph for new dependent relationships

Agents are eliminated with this process until only one agent remains. The last agent will possess the optimal joint payoff of the coordinating agents in terms of its own available actions. The optimal action of this agent is then propagated through the coordinated graph in the reverse order that agents were eliminated, giving each agent along the way enough information to decide what its own optimal action should be.

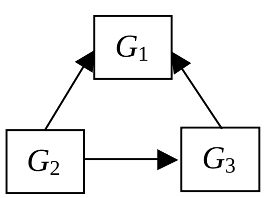
Here is a run-through of the example discussed in “Multi-robot decision making using coordination graphs,” with an elimination order of G_3, G_2, G_1.

Elimination of G_3:



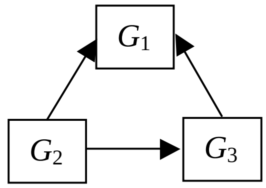
G_1	$\langle a_1 \wedge \neg a_3$	$: 4 \rangle$
	$\langle a_1 \wedge \neg a_2$	$: 5 \rangle$
G_2	$\langle \neg a_2$	$: 2 \rangle$
G_3	$\langle a_3 \wedge a_2$	$: 5 \rangle$

G_1 sends G_3 dependent rules:



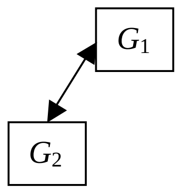
G_1	$\langle a_1 \wedge \neg a_2$	$: 5 \rangle$
G_2	$\langle \neg a_2$	$: 2 \rangle$
G_3	$\langle a_3 \wedge a_2$	$: 5 \rangle$
	$\langle a_1 \wedge \neg a_3$	$: 4 \rangle$

G_3 determines that the maximum local payoff is and distributes this to G_2:	$< a_2 : 5 >$ $< a_1 \wedge \neg a_2 : 4 >$



G_1	$< a_1 \wedge \neg a_2$	$: 5 >$
G_2	$< \neg a_2$	$: 2 >$
	$< a_2$	$: 5 >$
	$< a_1 \wedge \neg a_2$	$: 4 >$
G_3	$< a_3 \wedge a_2$	$: 5 >$
	$< a_1 \wedge \neg a_3$	$: 4 >$

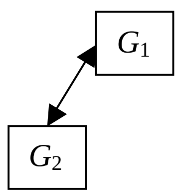
Dependencies are updated and G3 is removed:



G_1	$< a_1 \wedge \neg a_2$	$: 5 >$
G_2	$< \neg a_2$	$: 2 >$
	$< a_2$	$: 5 >$
	$< a_1 \wedge \neg a_2$	$: 4 >$

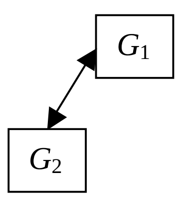
<u>*Elimination of G_2:*</u>

G_1 sends G_2 dependent rules:



G_1		
G_2	$< \neg a_2$	$: 2 >$
	$< a_2$	$: 5 >$
	$< a_1 \wedge \neg a_2$	$: 4 >$
	$< a_1 \wedge \neg a_2$	$: 5 >$

G_2 determines that the maximum local payoff is and distributes this to G_1	$< a_1 : 11 >$ $< \neg a_1 : 5 >$



G_1	$< a_1$	$: 11 >$
	$< \neg a_1$	$: 5 >$
G_2	$< \neg a_2$	$: 2 >$
	$< a_2$	$: 5 >$
	$< a_1 \wedge \neg a_2$	$: 4 >$
	$< a_1 \wedge \neg a_2$	$: 5 >$

G_2 is removed, leaving G_1 with the optimal decision of a_1.

G_1	$< a_1$	$: 11 >$
	$< \neg a_1$	$: 5 >$

G_1 distributes the decision of a_1 back to G_2. Knowing that a_1 is now true, G_2 chooses an optimal action of $\neg a_2$:

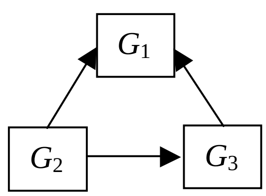
G_2	$< \neg a_2$	$: 2 >$	$\longrightarrow$	G_2	$< \neg a_2$	$: 11 >$
	$< a_2$	$: 5 >$			$< a_2$	$: 5 >$
	$< a_1 \wedge \neg a_2$	$: 4 >$				
	$< a_1 \wedge \neg a_2$	$: 5 >$				

G_2 distributes the decision of a_1 and $\neg a_2$ back to G_3. Knowing that a_1 is true and that a_2 is false, G_3 chooses an optimal action of $\neg a_3$:

G_3	$< a_3 \wedge a_2$	$: 5 >$	$\longrightarrow$	G_3	$< \neg a_3$	$: 4 >$
	$< a_1 \wedge \neg a_3$	$: 4 >$				

The variable elimination algorithm should produce the same results, regardless of order. Here is a run-through with an elimination order of G_1, G_3, G_2.

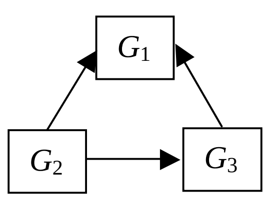
<u>*Elimination of G_1:*</u>



G_1	$< a_1 \wedge \neg a_3$	$: 4 >$
	$< a_1 \wedge \neg a_2$	$: 5 >$
G_2	$< \neg a_2$	$: 2 >$
G_3	$< a_3 \wedge a_2$	$: 5 >$

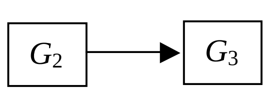
G_1 determines that the maximum local payoff is and distributes this to G_3:

$< \neg a_3$	$: 4 >$
$< \neg a_2$	$: 5 >$



G_1	$< a_1 \wedge \neg a_3$	$: 4 >$
	$< a_1 \wedge \neg a_2$	$: 5 >$
G_2	$< \neg a_2$	$: 2 >$
G_3	$< a_3 \wedge a_2$	$: 5 >$
	$< \neg a_3$	$: 4 >$
	$< \neg a_2$	$: 5 >$

G_1 is removed:

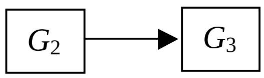


G_2	$< \neg a_2$	$: 2 >$
G_3	$< a_3 \wedge a_2$	$: 5 >$
	$< \neg a_3$	$: 4 >$
	$< \neg a_2$	$: 5 >$

Elimination of G_3:

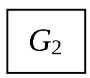
G_3 determines that the maximum local payoff is and distributes this to G_2

$< a_2$	$: 5 >$
$< \neg a_2$	$: 9 >$



G_2	$< \neg a_2$	$: 2 >$
	$< a_2$	$: 5 >$
	$< \neg a_2$	$: 9 >$
G_3	$< a_3 \wedge a_2$	$: 5 >$
	$< \neg a_3$	$: 4 >$
	$< \neg a_2$	$: 5 >$

G_3 is removed, leaving G_2 with the optimal decision of $\neg a_2$.



G_2	$< a_2$	$: 5 >$
	$< \neg a_2$	$: 11 >$

G_2 distributes the decision of $\neg a_2$ back to G_3. Knowing that a_2 is now false, G_3 chooses an optimal action of $\neg a_3$:

			→			
G_3	$< a_3 \wedge a_2$	$: 5 >$		G_3	$< \neg a_3$	$: 9 >$
	$< \neg a_3$	$: 4 >$			$< a_3$	$: 5 >$
	$< \neg a_2$	$: 5 >$				

G_3 distributes the decision of $\neg a_2$ and $\neg a_3$ back to G_1. Knowing that a_2 and a_3 are false, G_1 chooses an optimal action of a_1:

			→			
G_1	$< a_1 \wedge \neg a_3$	$: 4 >$		G_3	$< a_1$	$: 9 >$
	$< a_1 \wedge \neg a_2$	$: 5 >$				