CS378
Autonomous Multiagent Systems
Spring 2005

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Week 10a: Thursday, March 30th

Good Afternoon, Colleagues

Are there any questions?

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- how do you find NEQ?
- what if there are multiple NEQ?

Logistics

- Project progress reports due next week

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- Lots of game theory resources

Mixed strategy equilibrium

		Player 2	
		Action 1	Action 2
Player 1	Action 1	8,16	4,0
	Action 2	12,4	0,16

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- Player 2 must be indifferent between actions 1 and 2

Do actual numbers matter?

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Note: complexity unknown (likely in NP)

Tom's matrices

worker:

	Inspect	NoInspect
Work	WAGE-EFFORT_W	WAGE-EFFORT_W
Shirk	0	WAGE

supervisor:

	Inspect	NoInspect
Work	VALUE-WAGE-EFFORT_I	VALUE-WAGE
Shirk	-EFFORT_I	-WAGE

The nash equilibrium is as follows:

p(I) = EFFORT_W / WAGE

p(W) = EFFORT_I / WAGE

Correlated Equilibria

Sometimes mixing isn't enough: Bach/Stravinsky

		Wife	
		S	B
Me	S	2,1	0,0
	B	0,0	1,2

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Want only S,S or B,B - 50% each

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- When and where?
- What are the Nash equilibria?

Class Discussion

Luis Guimbarda on coalitions

Incomplete Information Games

- We each get one of 3 cards: 1,2,3
- If we both fold, we both lose nothing
- If one raises and one folds, the raiser gets 1
- If both raise, the one with the higher card gets 5
- Zero sum

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		Card ?	
		R	F
Card 3	R	5,-5	1,-1
	F	-1,1	0,0

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		Card ? R	Card ? F
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With more numbers and/or different payoffs, bluffing can be a part of the Nash Equilibrium

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- Change the **best response** of the other agent

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Threats slides

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  - General sum is the interesting case:
  - Learning in an environment with other, unknown, independent agents who may also be learning
  - Need to do well against some set of agents, never too poorly, and well against yourself.

Stochastic Games

- Tutorial slides