Topic 15
Implementing and Using Stacks

"stack n.
The set of things a person has to do in the future. "I haven't done it yet because every time I pop my stack something new gets pushed." If you are interrupted several times in the middle of a conversation, "My stack overflowed" means "I forget what we were talking about."

-The Hacker's Dictionary

Friedrich L. Bauer
German computer scientist who proposed "stack method of expression evaluation" in 1955.
Sharper Tools

Lists

Stacks
Stacks

- Access is allowed only at one point of the structure, normally termed the *top* of the stack
  - access to the most recently added item only
- Operations are limited:
  - push (add item to stack)
  - pop (remove top item from stack)
  - top (get top item without removing it)
  - isEmpty
- Described as a "Last In First Out" (LIFO) data structure
Stack Operations

Assume a simple stack for integers.

```java
Stack<Integer> s = new Stack<Integer>();
s.push(12);
s.push(4);
s.push( s.top() + 2 );
s.pop();
s.pop();
s.push( s.top() );
//what are contents of stack?
```
Stack Operations

Write a method to print out contents of stack in reverse order.
Uses of Stacks

- The runtime stack used by a process (running program) to keep track of methods in progress
- Search problems
- Undo, redo, back, forward
What is Output?

Stack<Integer> s = new Stack<Integer>();
// put stuff in stack
for(int i = 0; i < 5; i++)
    s.push(i);
// Print out contents of stack
// while emptying it.
// Assume there is a size method.
for(int i = 0; i < s.size(); i++)
    System.out.print(s.pop() + " ");

A  0 1 2 3 4
B  4 3 2 1 0
C  4 3 2
D  2 3 4
E  No output due to runtime error
Corrected Version

Stack<Integer> s = new Stack<Integer>();
// put stuff in stack
for(int i = 0; i < 5; i++)
    s.push( i );
// print out contents of stack
// while emptying it
int limit = s.size();
for(int i = 0; i < limit; i++)
    System.out.print( s.pop() + " ");
// or
// while( !s.isEmpty() )
//     System.out.println( s.pop() );
Implementing a stack

- need an underlying collection to hold the elements of the stack
- 2 obvious choices
  - array (native or ArrayList)
  - linked list

Adding a *layer of abstraction*. A big idea.

array implementation

linked list implementation
Applications of Stacks
Mathematical Calculations

- What does $3 + 2 \times 4$ equal?
  $2 \times 4 + 3$?
  $3 \times 2 + 4$?

- The precedence of operators affects the order of operations.

- A mathematical expression cannot simply be evaluated left to right.

- A challenge when evaluating a program.

- *Lexical analysis* is the process of interpreting a program.

What about $1 - 2 - 4^5 \times 3 \times 6 / 7^2^3$
Infix and Postfix Expressions

- The way we are use to writing expressions is known as infix notation.
- Postfix expression does not require any precedence rules.
- \(3 2 \times 1 +\) is postfix of \(3 \times 2 + 1\).
- Evaluate the following postfix expressions and write out a corresponding infix expression:
  - \(2 3 2 4 \times + \times\)
  - \(1 2 3 4 \wedge \times +\)
  - \(1 2 - 3 2 \wedge 3 \times 6 / +\)
  - \(2 5 \wedge 1 -\)
Clicker Question 2

What does the following postfix expression evaluate to?

6 3 2 + *

A. 18
B. 36
C. 24
D. 11
E. 30
Evaluation of Postfix Expressions

- Easy to do with a stack
- given a proper postfix expression:
  - get the next token
  - if it is an operand push it onto the stack
  - else if it is an operator
    - pop the stack for the right hand operand
    - pop the stack for the left hand operand
    - apply the operator to the two operands
    - push the result onto the stack
  - when the expression has been exhausted the result is the top (and only element) of the stack
Infix to Postfix

Convert the following equations from infix to postfix:

$2 \, ^ \, \, ^ \, \, ^ \, 3 \, + \, 5 \, \times \, 1$

$11 \, + \, 2 \, - \, 1 \, \times \, 3 \, / \, 3 \, + \, 2 \, \times \, 2 \, / \, 3$

Problems:

- Negative numbers?
- Parentheses in expression
Infix to Postfix Conversion

- Requires operator precedence parsing algorithm
  - parse v. To determine the syntactic structure of a sentence or other utterance

Operands: add to expression

Close parenthesis: pop stack symbols until an open parenthesis appears

Operators:
  - Have an on stack and off stack precedence
  - Pop all stack symbols until a symbol of lower precedence appears. Then push the operator

End of input: Pop all remaining stack symbols and add to the expression
Simple Example

Infix Expression: $3 + 2 * 4$

PostFix Expression:

Operator Stack:

<table>
<thead>
<tr>
<th>Precedence Table</th>
</tr>
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<tbody>
<tr>
<td>Symbol</td>
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Simple Example

Infix Expression: \( + 2 * 4 \)

PostFix Expression: \( 3 \)

Operator Stack:

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Simple Example

Infix Expression: \(2 \times 4\)

PostFix Expression: 3

Operator Stack: +

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Simple Example

Infix Expression: * 4

PostFix Expression: 3 2

Operator Stack: +

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Simple Example

Infix Expression: 4
PostFix Expression: 3 2
Operator Stack: + *

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Simple Example

Infix Expression:

PostFix Expression: 3 2 4

Operator Stack: + *

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Infix Expression:
PostFix Expression: \[ 3 \ 2 \ 4 \ * \]
Operator Stack: \[ + \]

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Simple Example

Infix Expression:

PostFix Expression: \[ 3 \ 2 \ 4 \ * \ + \]

Operator Stack:

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Example

$11 + 2^4^3 - ( (4 + 5) * 6 )^2$

Show algorithm in action on above equation
Balanced Symbol Checking

- In processing programs and working with computer languages there are many instances when symbols must be balanced \{ \}, [ ], ( )

A stack is useful for checking symbol balance. When a closing symbol is found it must match the most recent opening symbol of the same type.

- Applicable to checking html and xml tags!
Algorithm for Balanced Symbol Checking

- Make an empty stack
- read symbols until end of file
  - if the symbol is an opening symbol push it onto the stack
  - if it is a closing symbol do the following
    - if the stack is empty report an error
    - otherwise pop the stack. If the symbol popped does not match the closing symbol report an error
- At the end of the file if the stack is not empty report an error
Algorithm in practice

- \( \text{list}[i] = 3 \times (44 - \text{method( foo( \text{list}[2 \times (i + 1) + \text{foo( \text{list}[i - 1] ) ) / 2 \times )} \right) - \text{list[ method(\text{list}[0])]}; \)

- Complications
  - when is it not an error to have non matching symbols?

- Processing a file
  - *Tokenization*: the process of scanning an input stream. Each independent chunk is a token.

- Tokens may be made up of 1 or more characters