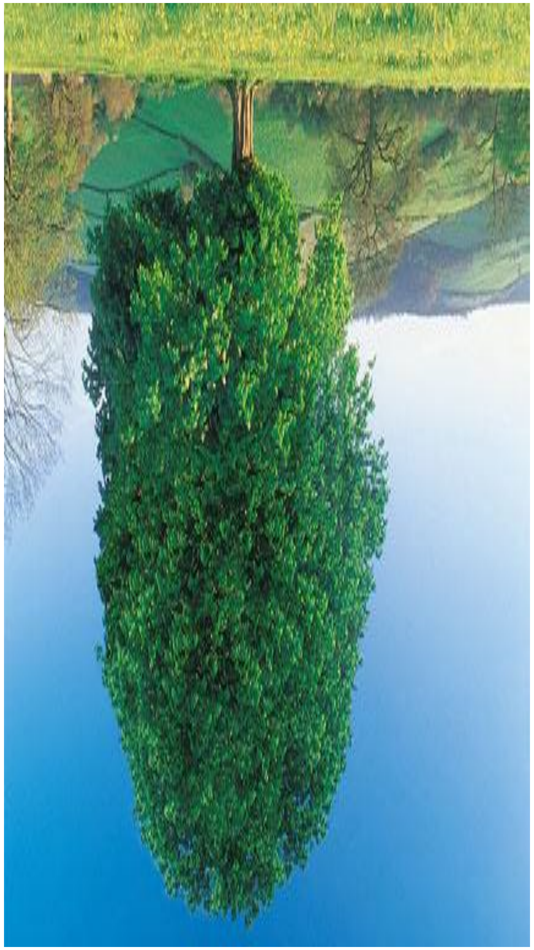
Topic 18
Binary Trees

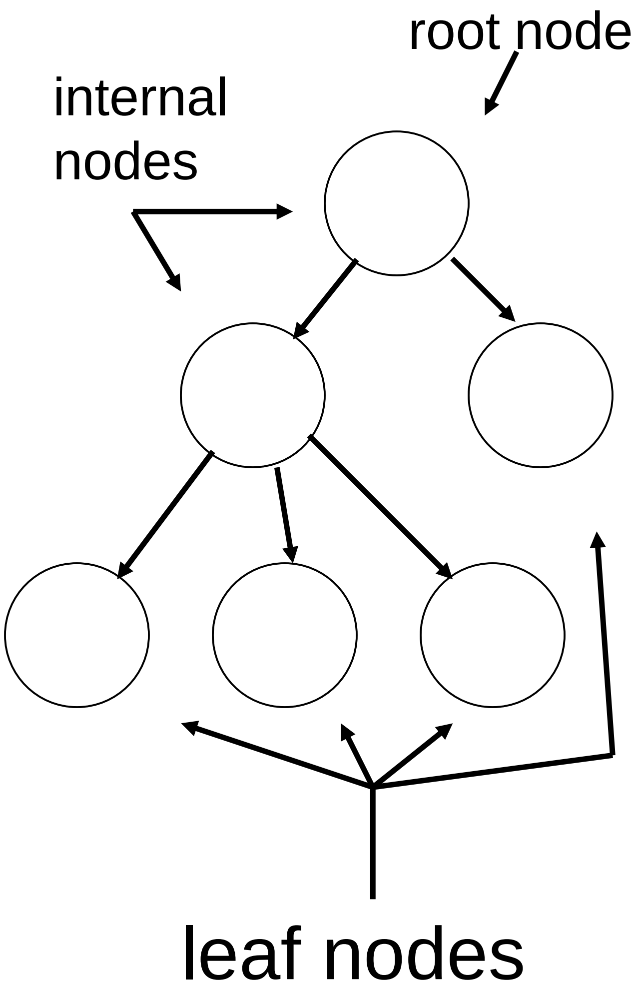
"A tree may grow a thousand feet tall, but its leaves will return to its roots."

-Chinese Proverb



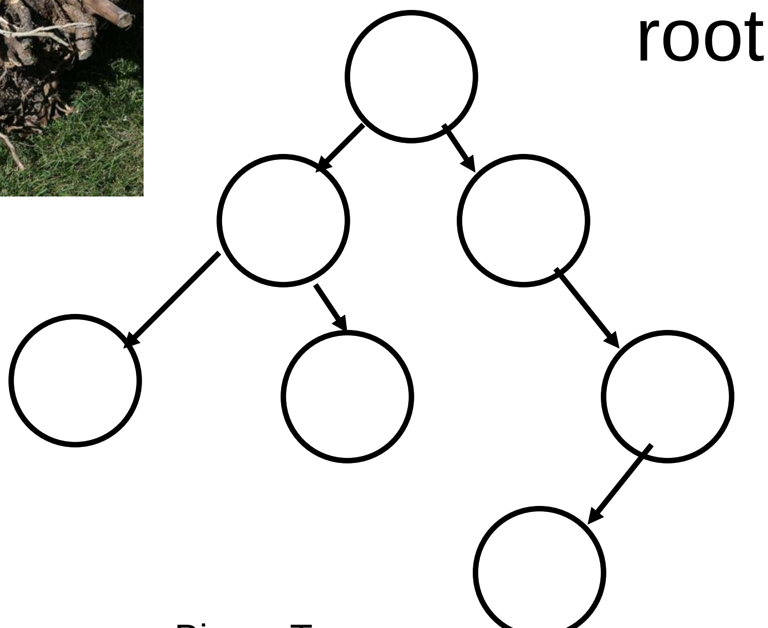
Definitions

- A *tree* data structure
  - one entry point, the ***root***
  - Each node is either a ***leaf*** or an *internal node*
  - An internal node has 1 or more ***children***, nodes that can be reached directly from that internal node.
  - The internal node is said to be the ***parent*** of its child nodes



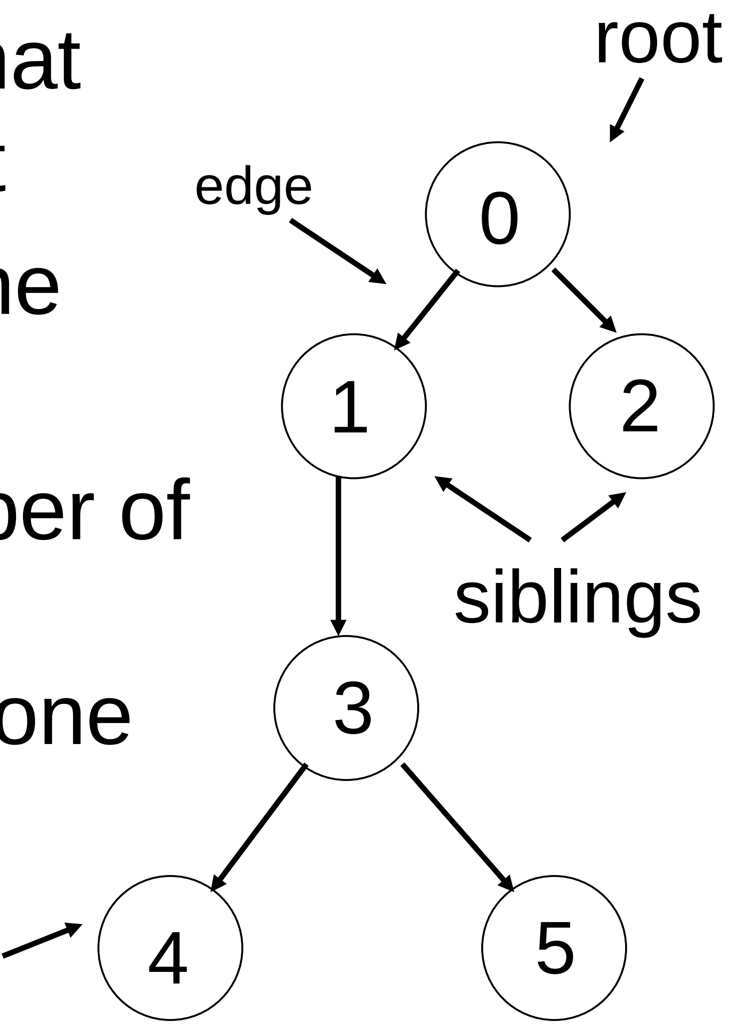
Properties of Trees

- Only access point is the root
- All nodes, except the root, have one parent
  – like the inheritance hierarchy in Java
- Traditionally trees drawn upside down



Properties of Trees and Nodes

- *siblings:* two nodes that have the same parent
- *edge:* the link from one node to another
- *path length:* the number of edges that must be traversed to get from one node to another

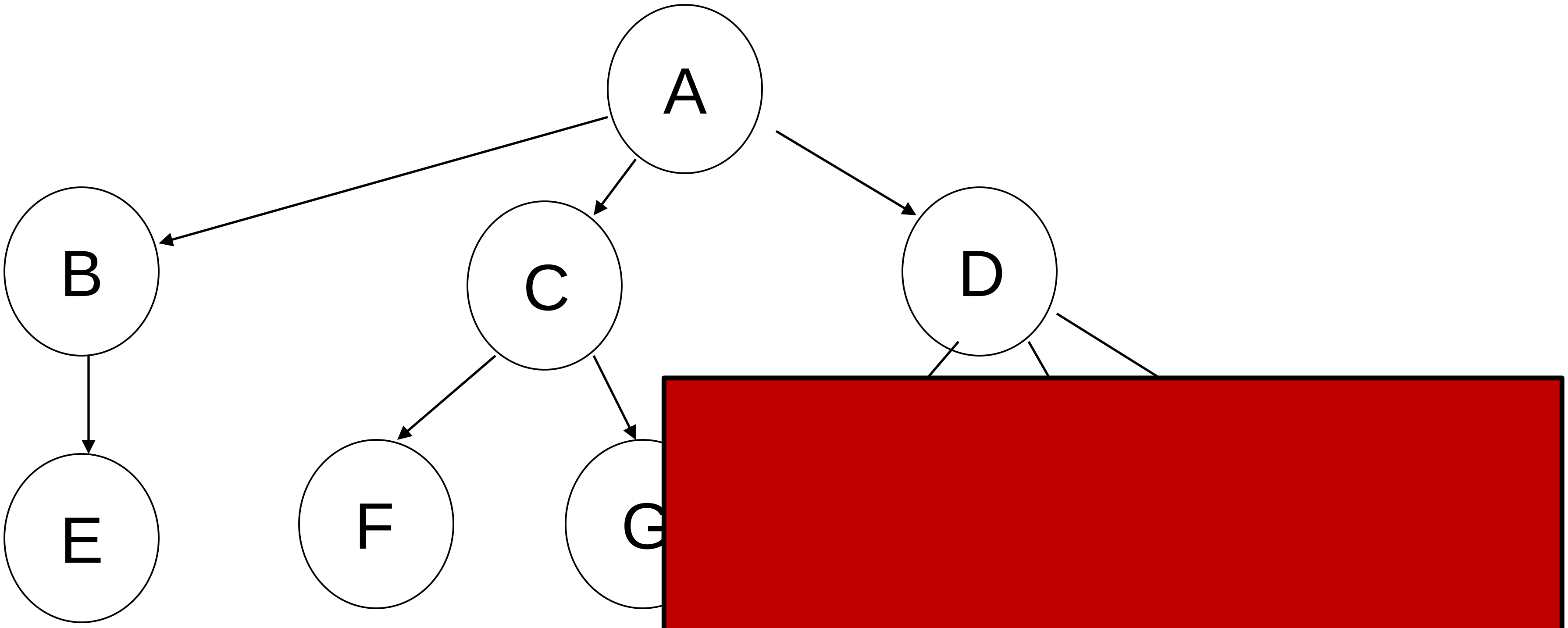


path length from root to this node is 3

More Properties of Trees

- ***depth**:* the path length from the root of the tree to this node
- ***height of a node**:* The maximum distance (path length) of any leaf from this node
  - a leaf has a height of 0
  - the height of a tree is the height of the root of that tree
  - what if only one node, the root?
  - what if empty? Discover OptionalInt
- *descendants*: any nodes that can be reached via 1 or more edges from this node
- *ancestors:* any nodes for which this node is a descendant

Tree Visualization



Clicker 1

- What is the depth of the node that contains M on the previous slide?

A. 0

B. 1

C. 2

D. 3

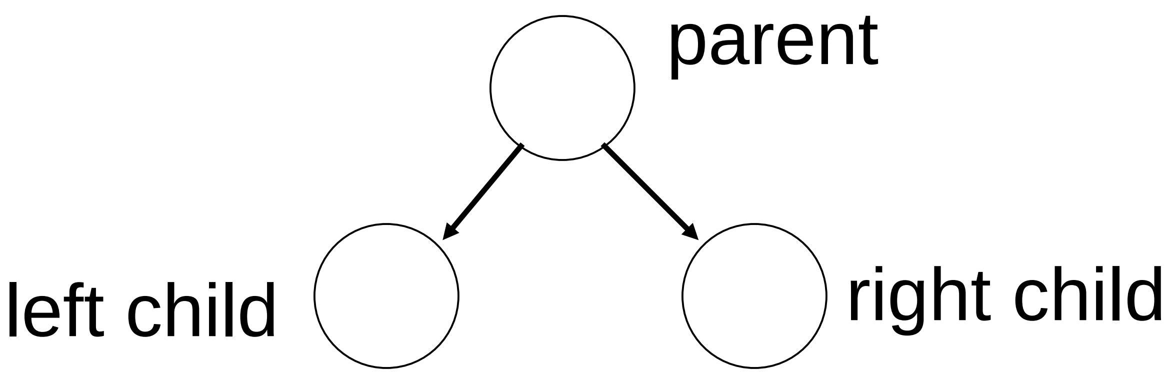
E. 4

Clicker 2 - Same tree, same choices

What is the height of the node that contains D?

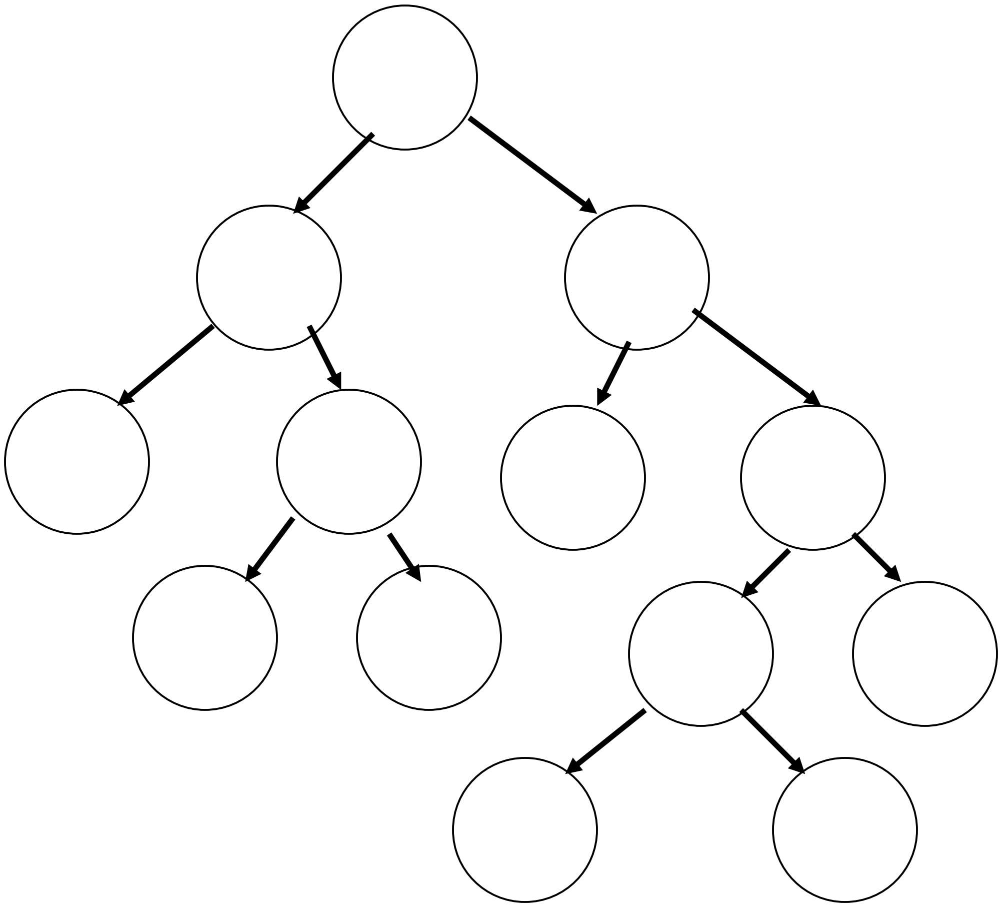
Binary Trees

- There are many variations on trees but we will start with *binary trees*
- *binary tree:* each node has at most two children
  - the possible children are usually referred to as the left child and the right child



Full Binary Tree

- *full binary tree:* a binary tree in which each node has 2 or 0 children



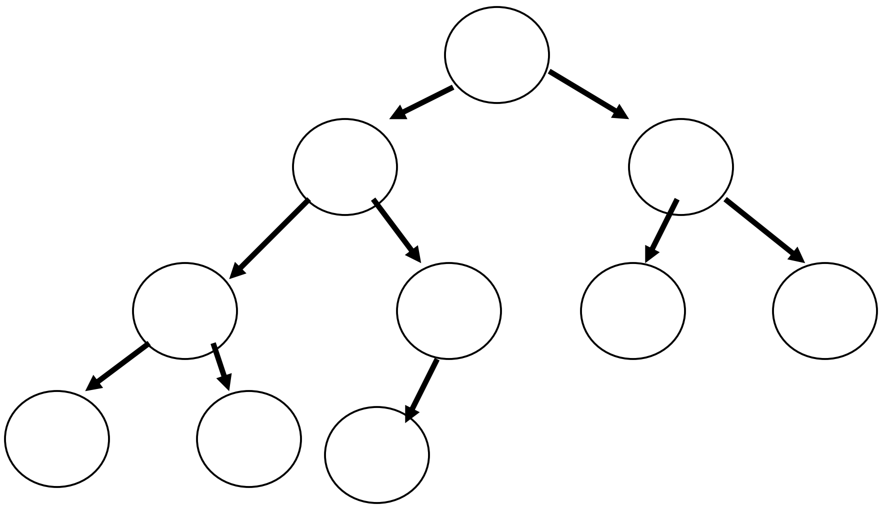
Clicker 3

- What is the maximum height of a full binary tree with 11 nodes?

A. 3
B. 5
C. 7
D. 10
E. Not possible to have full binary tree with 11 nodes.

Complete Binary Tree

- *complete binary tree:* a binary tree in which every level, except possibly the deepest is completely filled. At depth n, the height of the tree, all nodes are as far left as possible



Where would the next node go to maintain a complete tree?

Clicker 4

- What is the height of a complete binary tree that contains N nodes?

A. O(1)

B. O(logN)

C. $O(N^{1/2})$

D. O(N)

E. O(NlogN)

- Recall, order can be applied to any function. It doesn't just apply to running time.

Perfect Binary Tree

- *perfect binary tree:* a binary tree with all leaf nodes at the same depth. All internal nodes have exactly two children.
- a perfect binary tree has the maximum number of nodes for a given height
- a perfect binary tree has $(2^{(n+1)} - 1)$ nodes where n is the height of the tree
  - height = 0 -> 1 node
  - height = 1 -> 3 nodes
  - height = 2 -> 7 nodes
  - height = 3 -> 15 nodes

A Binary Node class

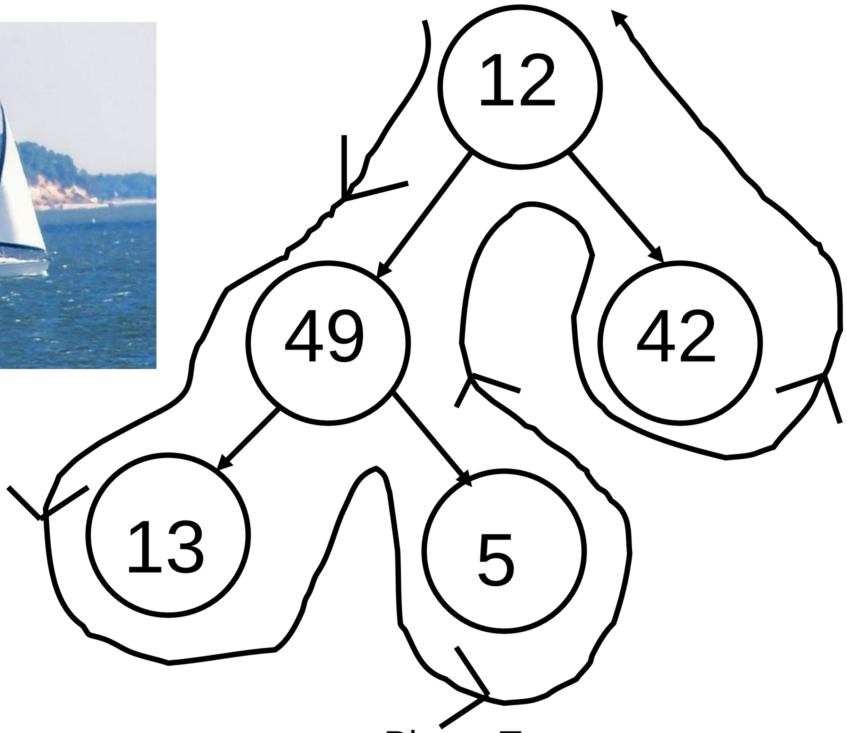
```
public class Bnode<E> {
     private E myData;
     private Bnode<E> myLeft;
     private Bnode<E> myRight;

     public BNode();
     public BNode(Bnode<E> left, E data,
          Bnode<E> right)
     public E getData()
     public Bnode<E> getLeft()
     public Bnode<E> getRight()

     public void setData(E data)
     public void setLeft(Bnode<E> left)
     public void setRight(Bnode<E> right)
}
```

Binary Tree Traversals

- Many algorithms require all nodes of a binary tree be visited and the contents of each node processed or examined.
- There are 4 traditional types of traversals
  - preorder traversal: process the root, then process all sub trees (left to right)
  - in order traversal: process the left sub tree, process the root, process the right sub tree
  - post order traversal: process the left sub tree, process the right sub tree, then process the root
  - level order traversal: starting from the root of a tree, process all nodes at the same depth from left to right, then proceed to the nodes at the next depth.

Results of Traversals

- To determine the results of a traversal on a given tree draw a path around the tree.
  - start on the left side of the root and trace around the tree. The path should stay close to the tree.

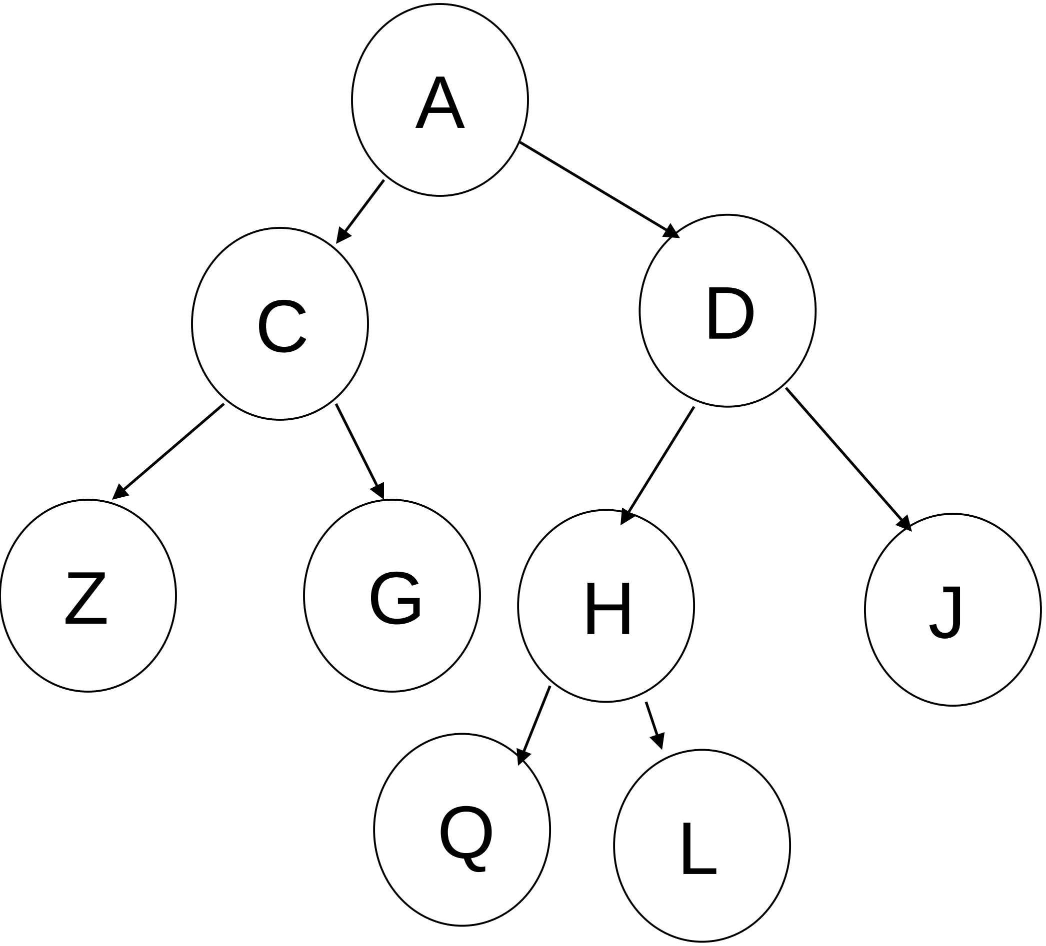


pre order: process when pass down left side of node
12 49 13 5 42

in order: process when pass underneath node
13 49 5 12 42

post order: process when pass up right side of node
13 5 49 42 12

Clicker 5 - Tree Traversals



What is a the result of a post order traversal of the tree to the left?

A. Z C G A Q H L D J
B. Z G C Q L H J D A
C. A C Z G D H Q L J
D. A C D Z G H J Q L
E. None of these

Implement Traversals

- Implement preorder, inorder, and post order traversal
  - Big O time and space?
- Implement a level order traversal using a queue
  - Big O time and space?
- Implement a level order traversal without a queue
  - target depth

Breadth First Search
Depth First Search

- from NIST - DADS
- **breadth first search:** Any search algorithm that considers neighbors of a *vertex* (node), that is, outgoing *edges* (links) of the vertex's predecessor in the search, before any outgoing edges of the vertex
- **depth first search:** Any search algorithm that considers outgoing *edges* (links of *children*) of a *vertex* (node) before any of the vertex's (node) *siblings*, that is, outgoing edges of the vertex's predecessor in the search. Extremes are searched first.

Clicker 6

- Which traversal of a tree is a breadth first search?

A. Level order traversal

B. Pre order traversal

C. In order traversal

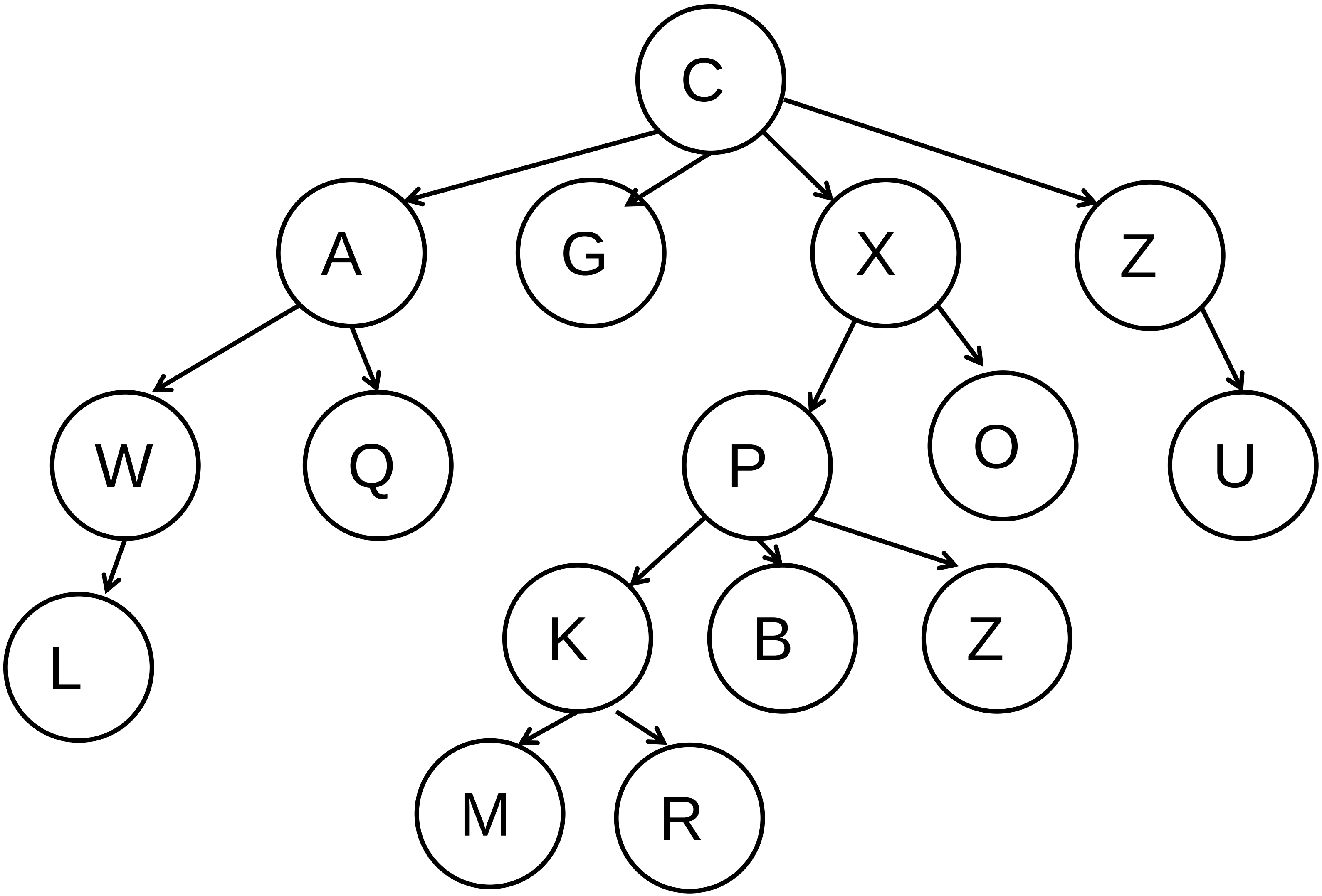
D. Post order traversal

E. More than one of these

Breadth First

- A level order traversal of a tree could be used as a breadth first search
- Search all nodes in a level before going down to the next level

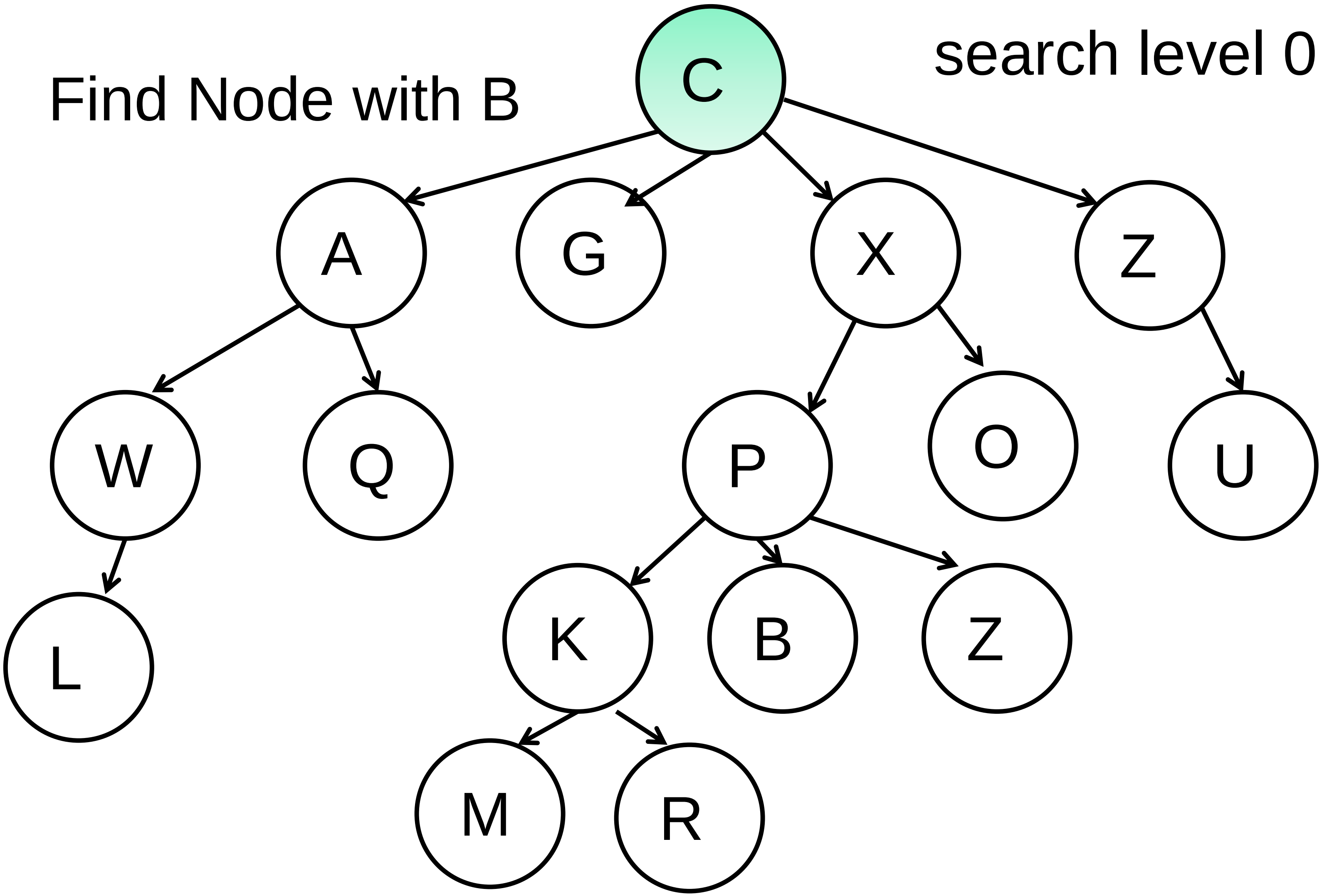
Breadth First Search of Tree



Breadth First Search

Find Node with B

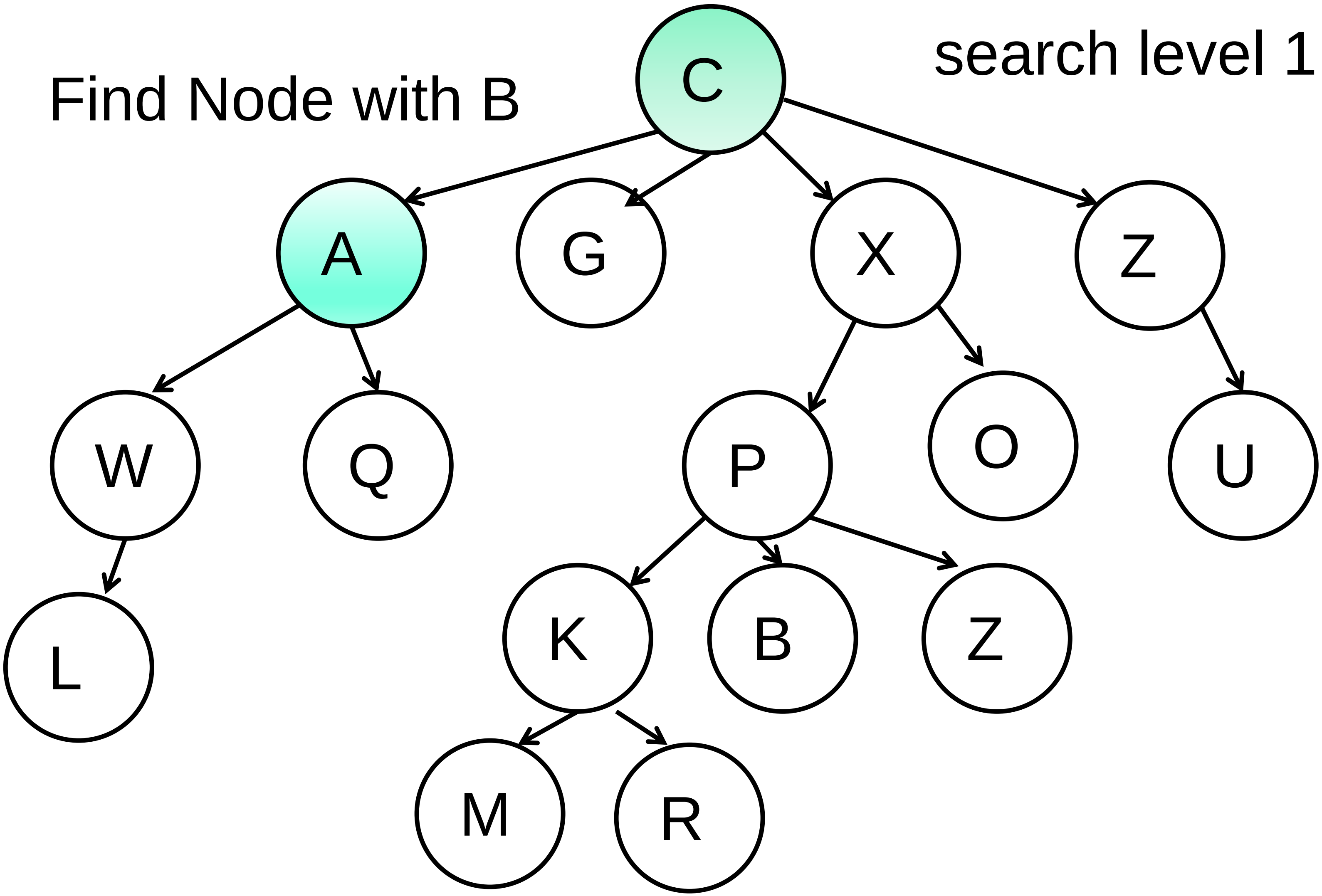
search level 0 first



Breadth First Search

Find Node with B

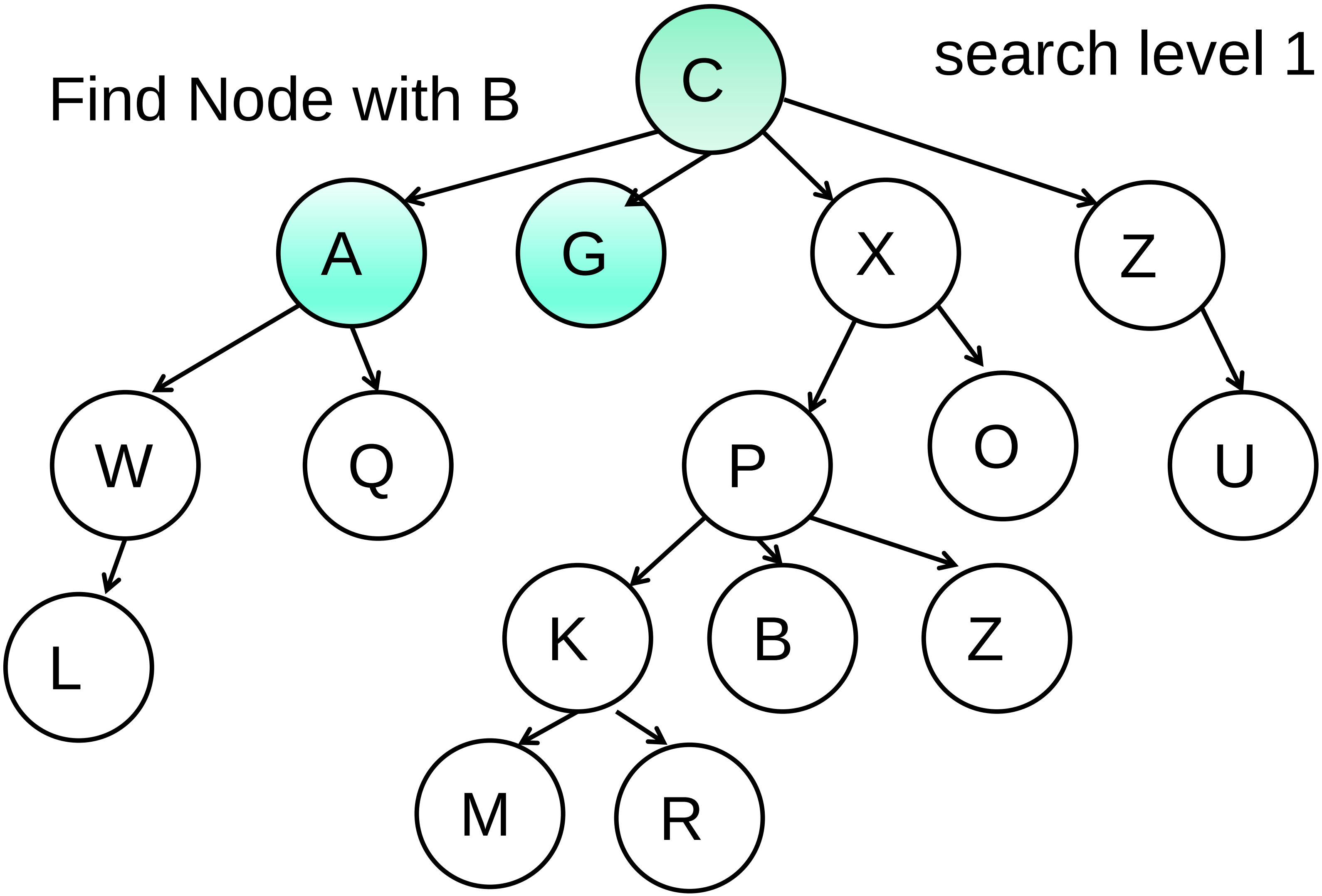
search level 1



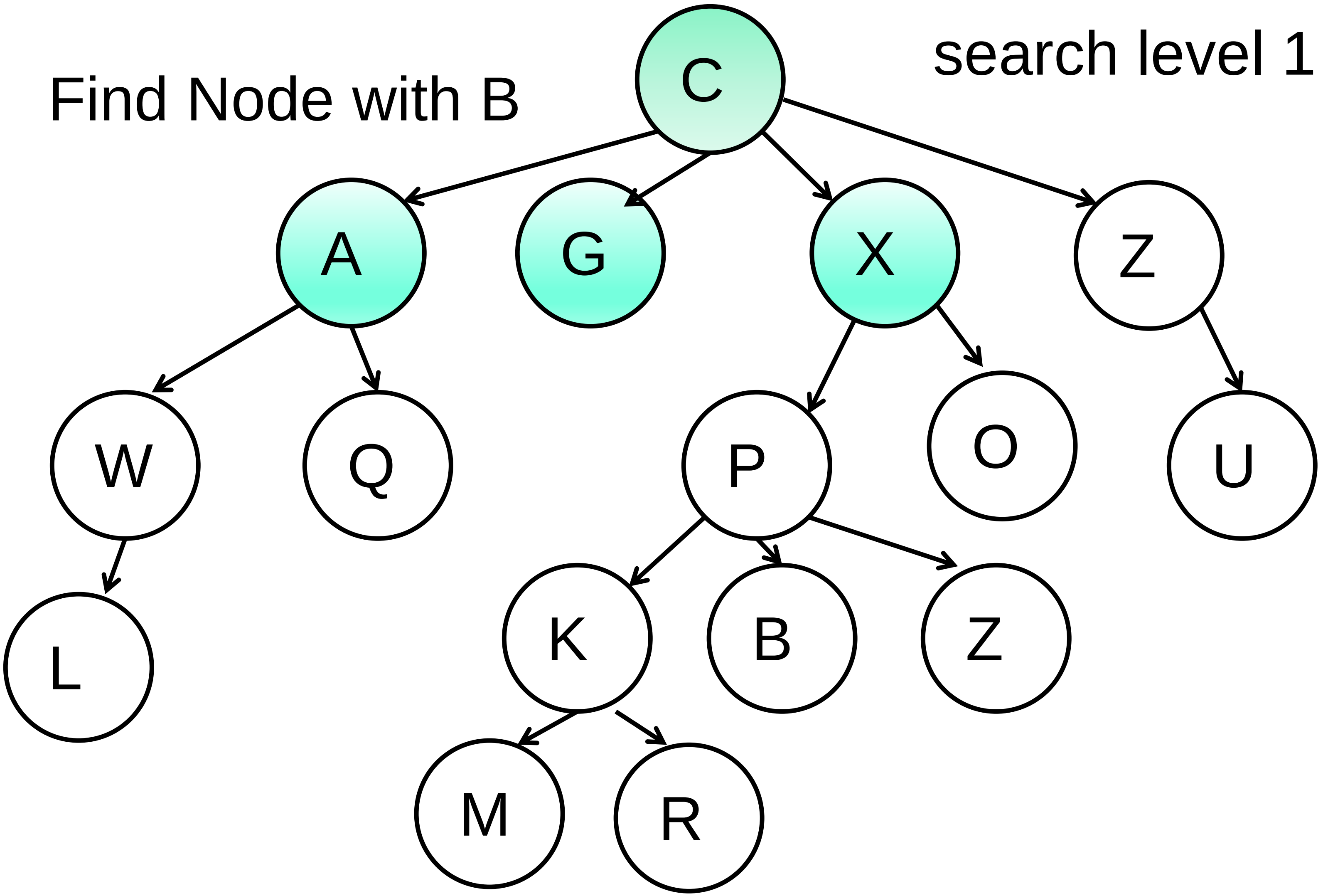
Breadth First Search

Find Node with B

search level 1



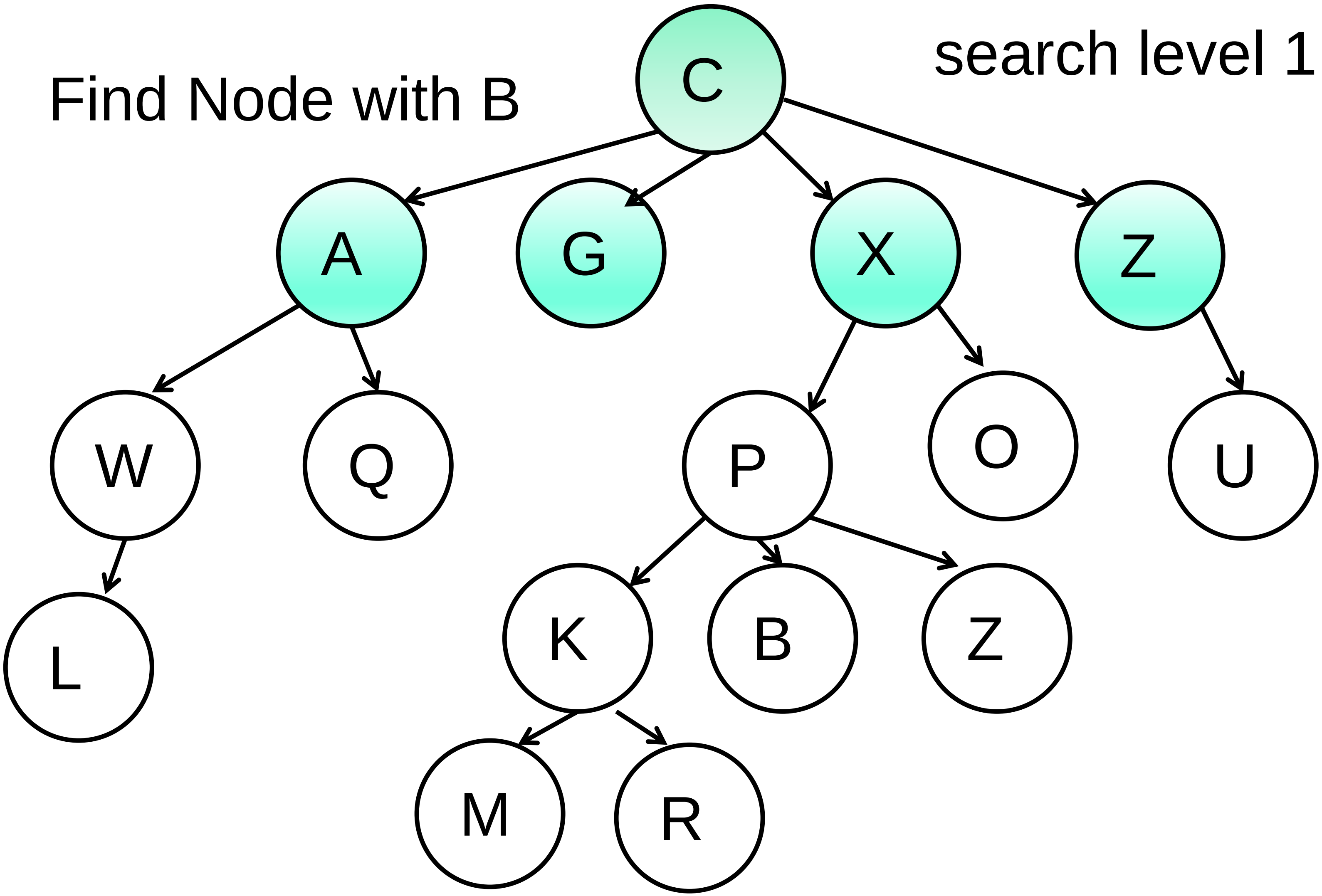
Breadth First Search



Breadth First Search

Find Node with B

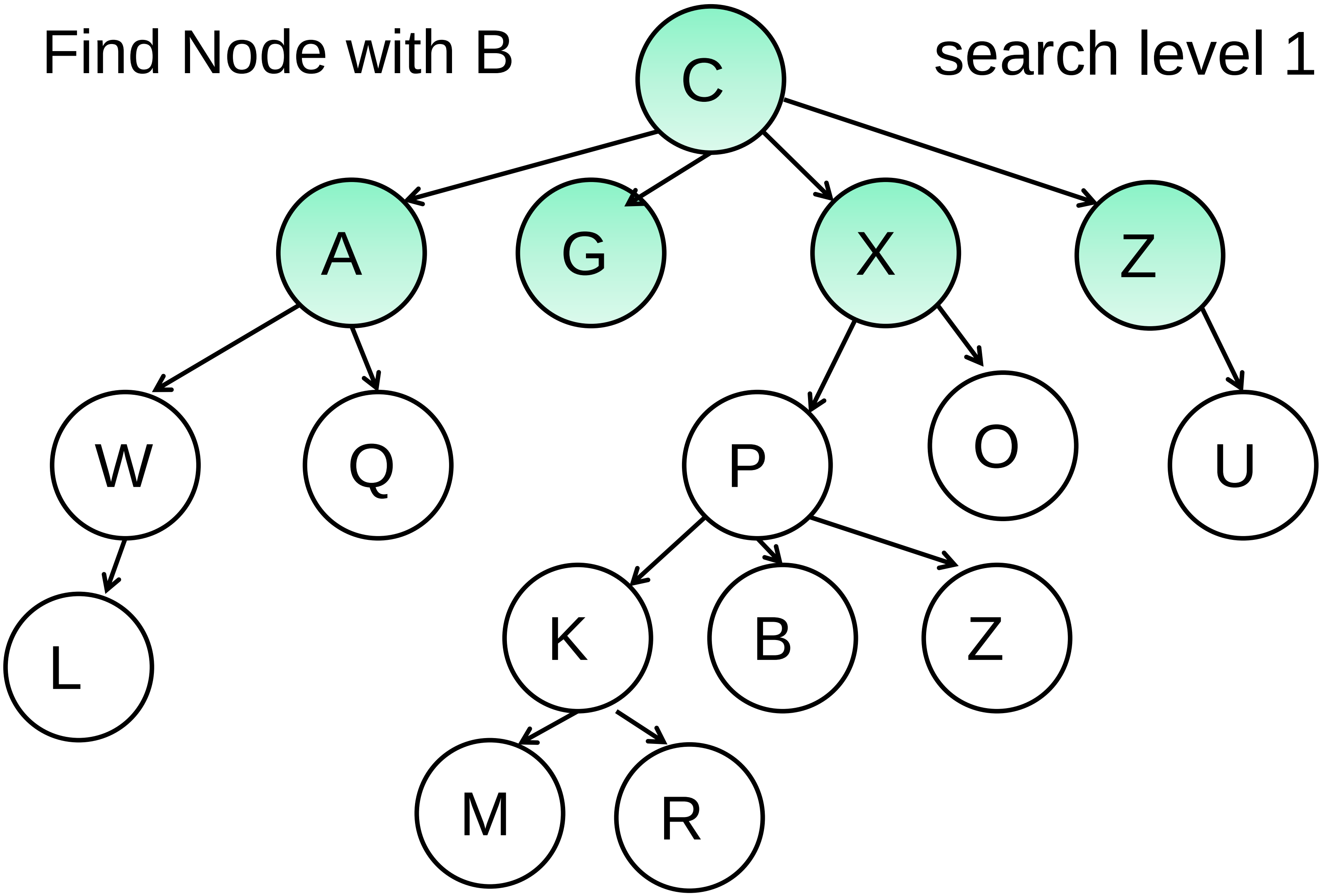
search level 1



Breadth First Search

Find Node with B

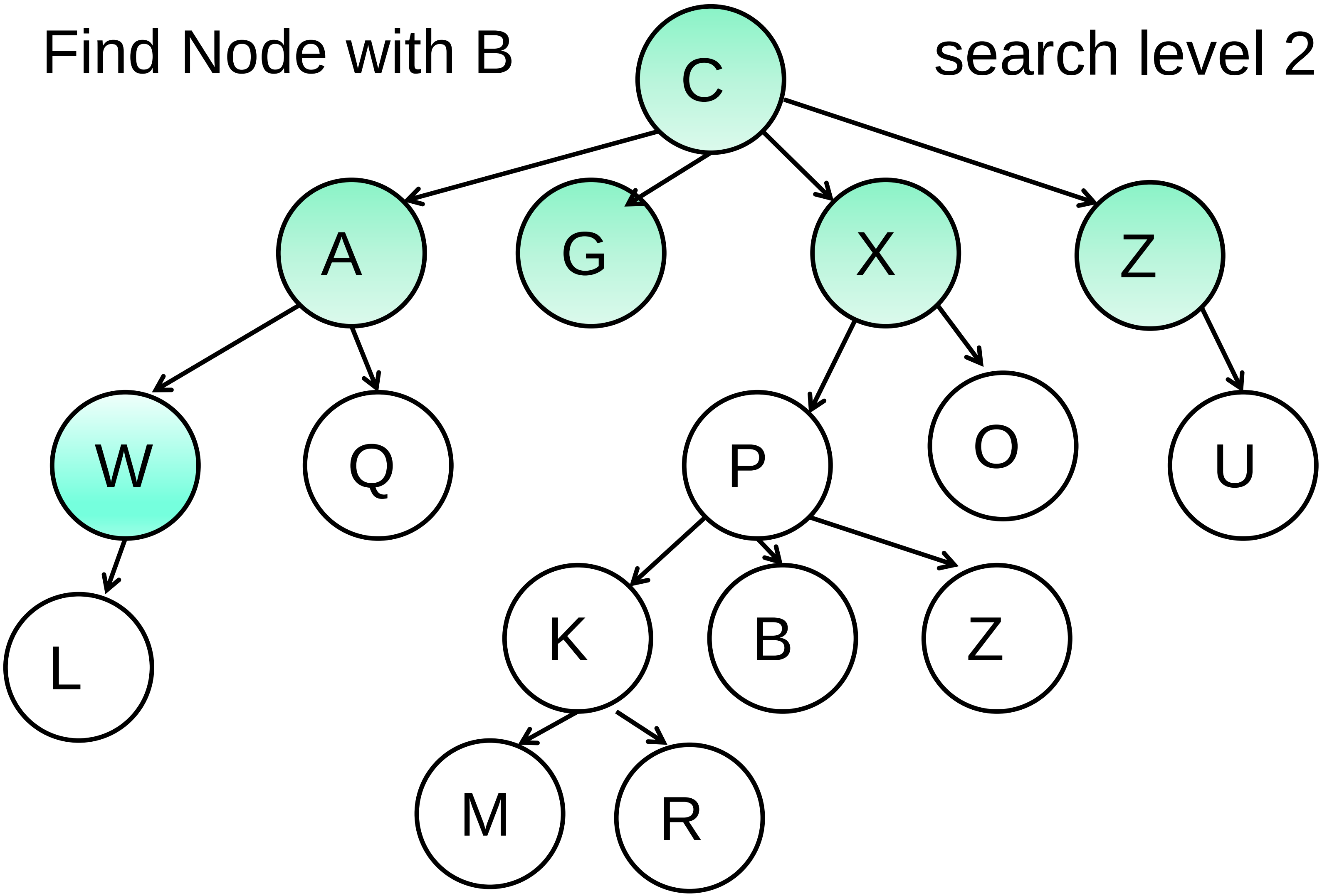
search level 1 next



Breadth First Search

Find Node with B

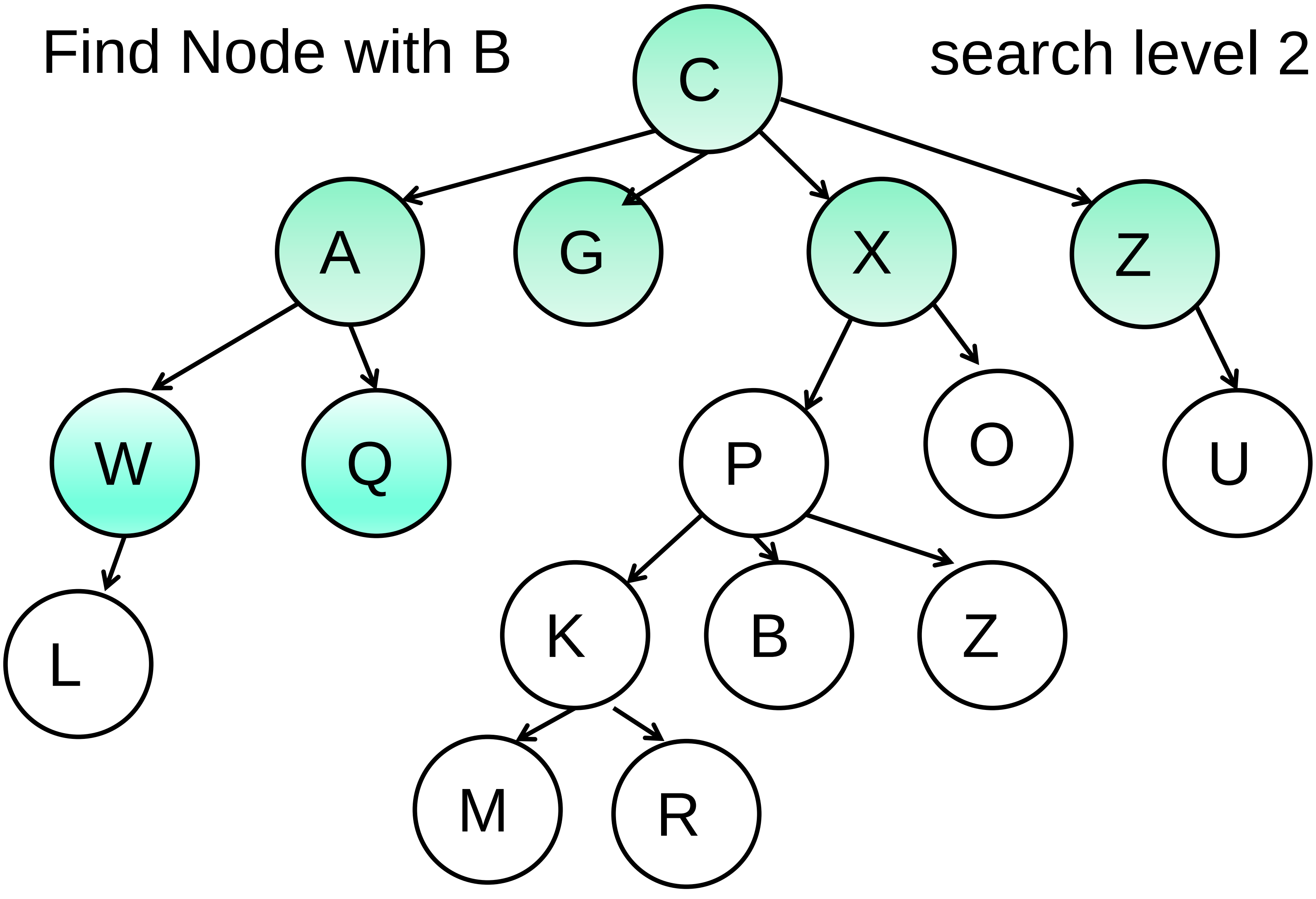
search level 2 next



Breadth First Search

Find Node with B

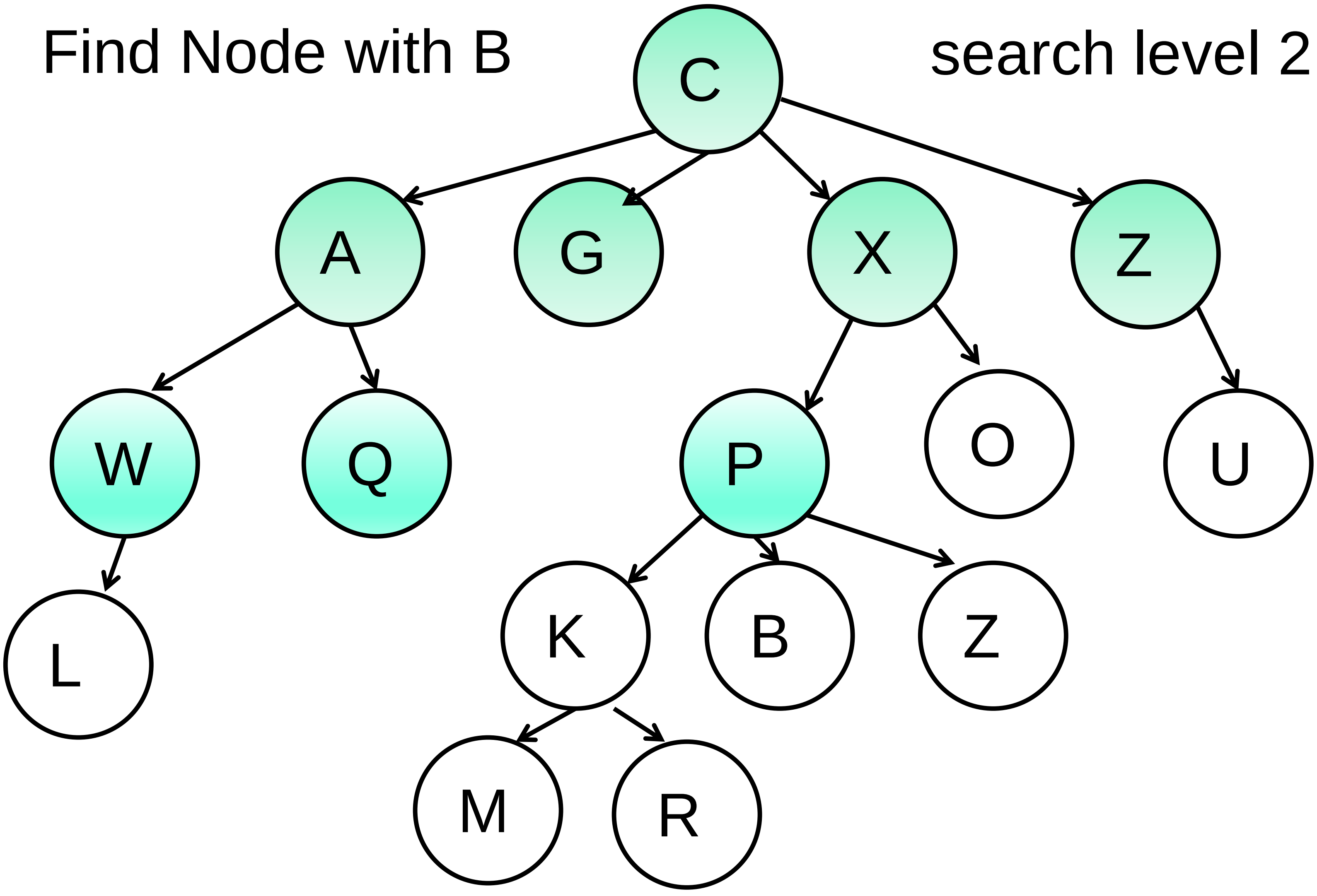
search level 2 next



Breadth First Search

Find Node with B

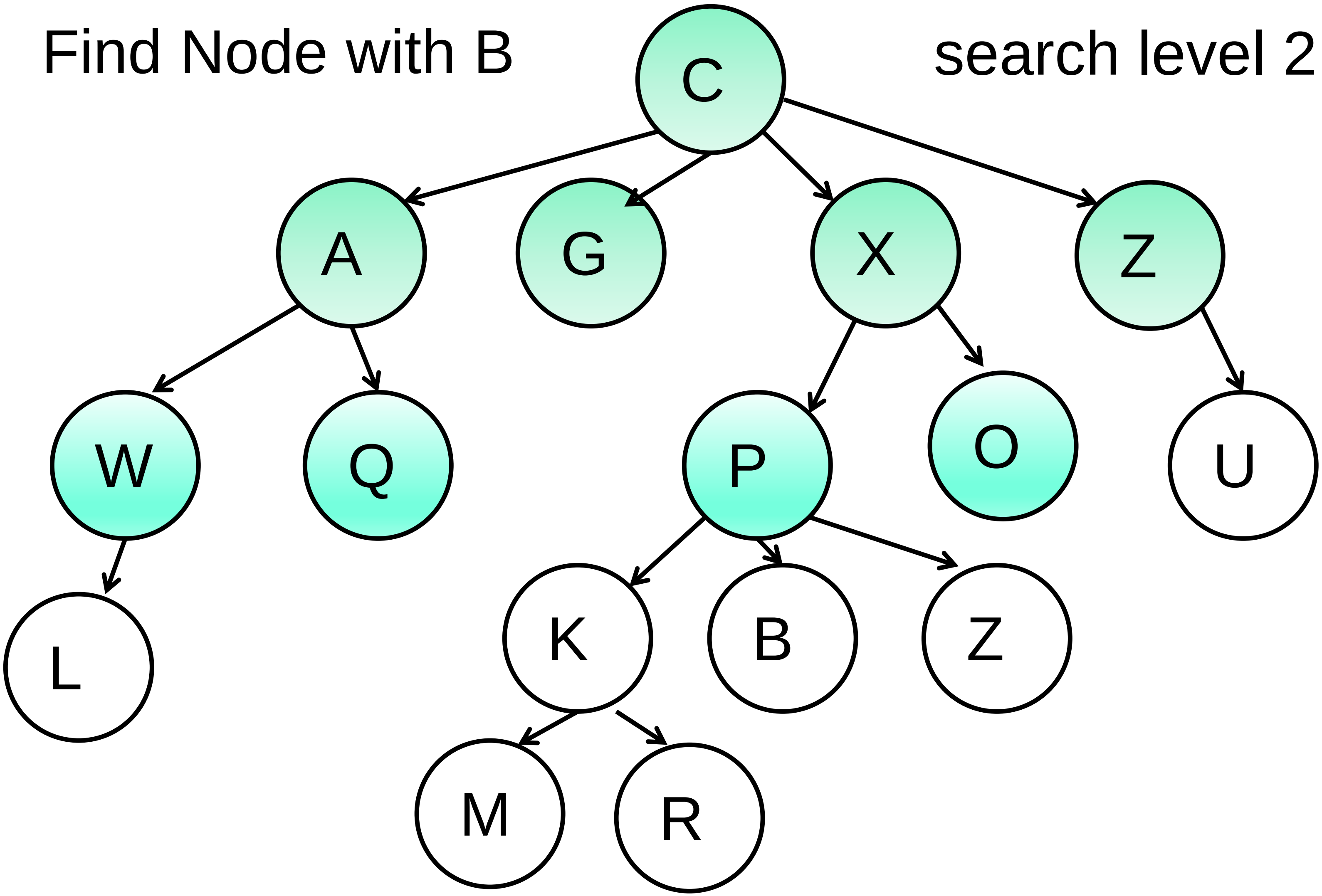
search level 2 next



Breadth First Search

Find Node with B

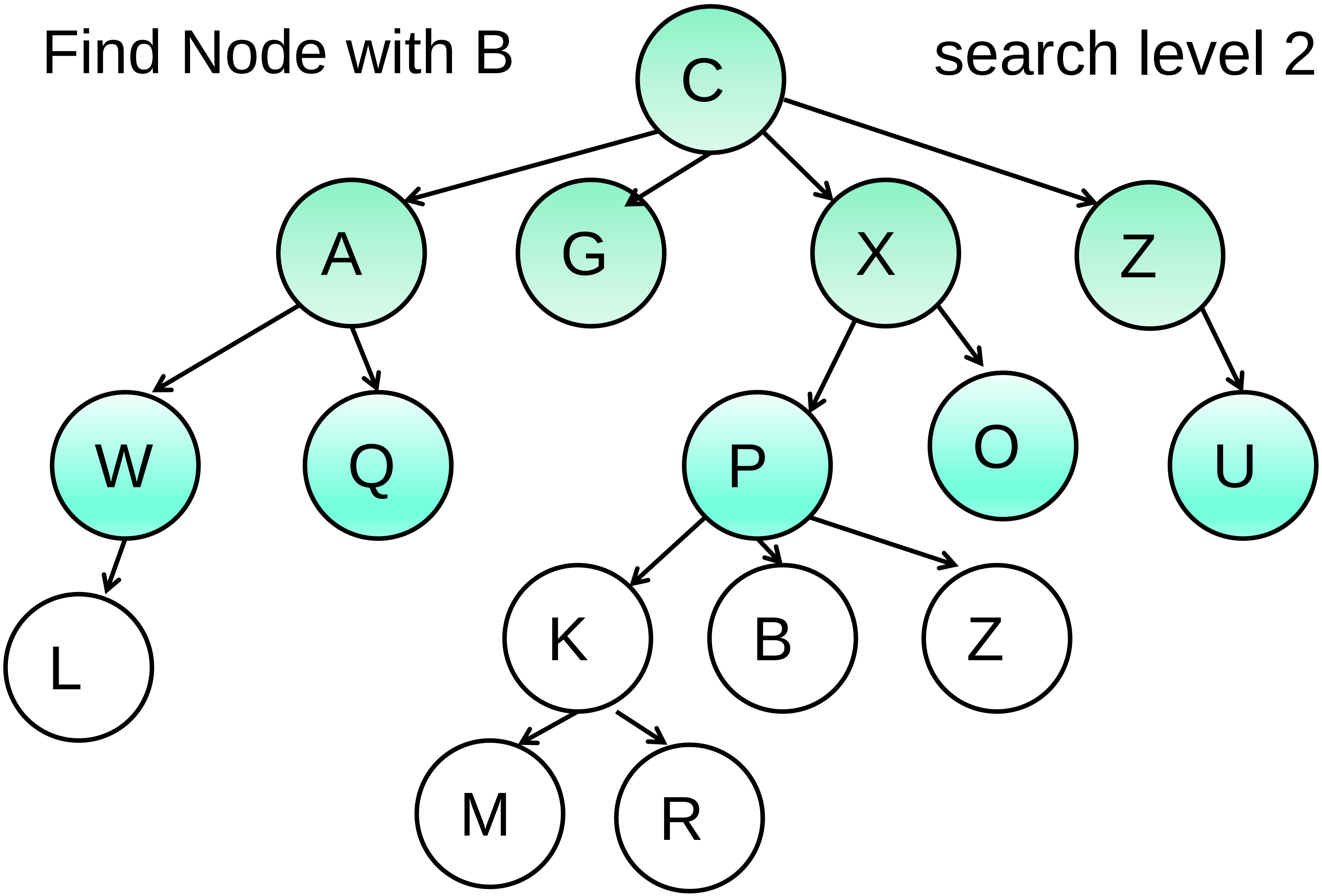
search level 2 next



Breadth First Search

Find Node with B

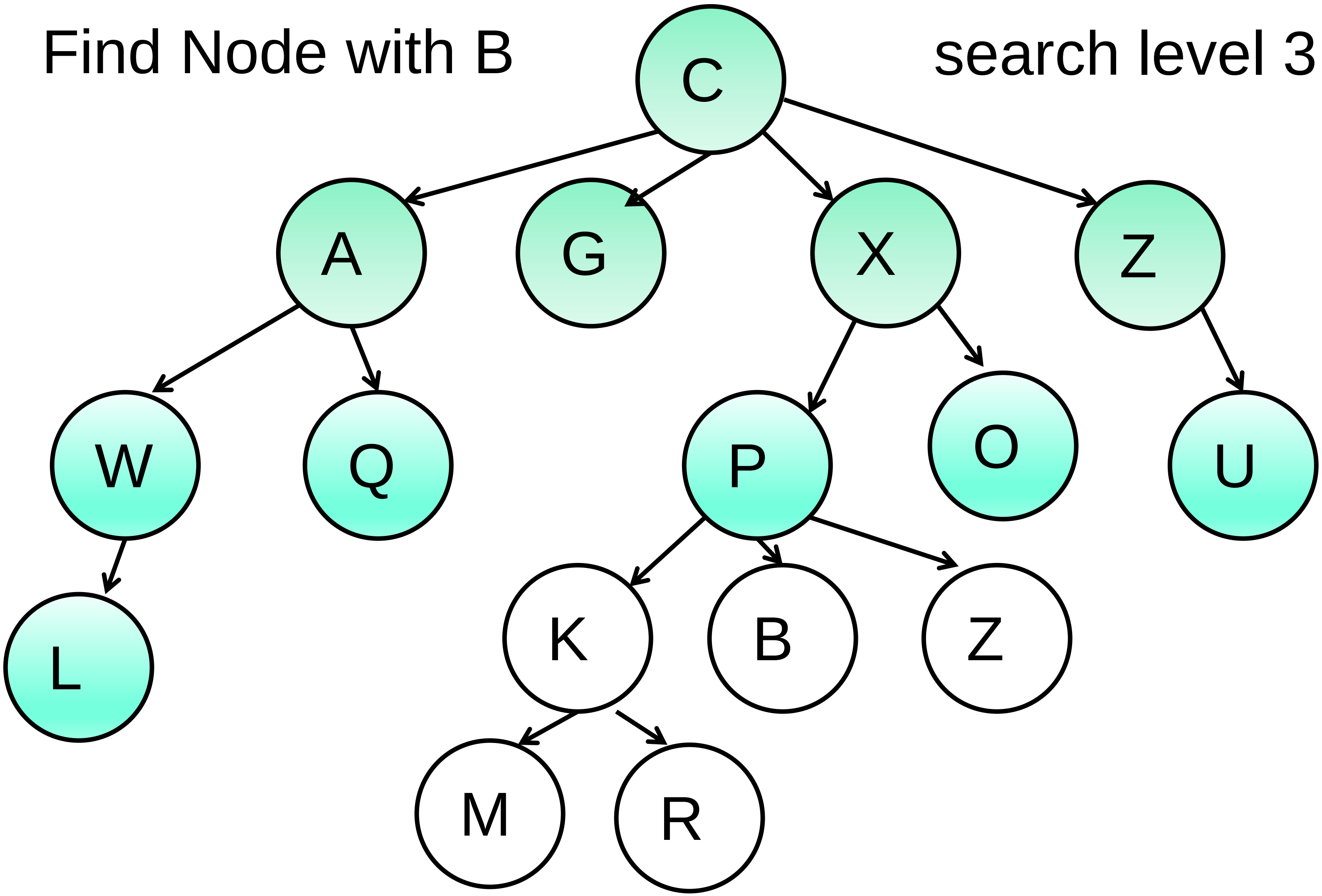
search level 2 next



Breadth First Search

Find Node with B

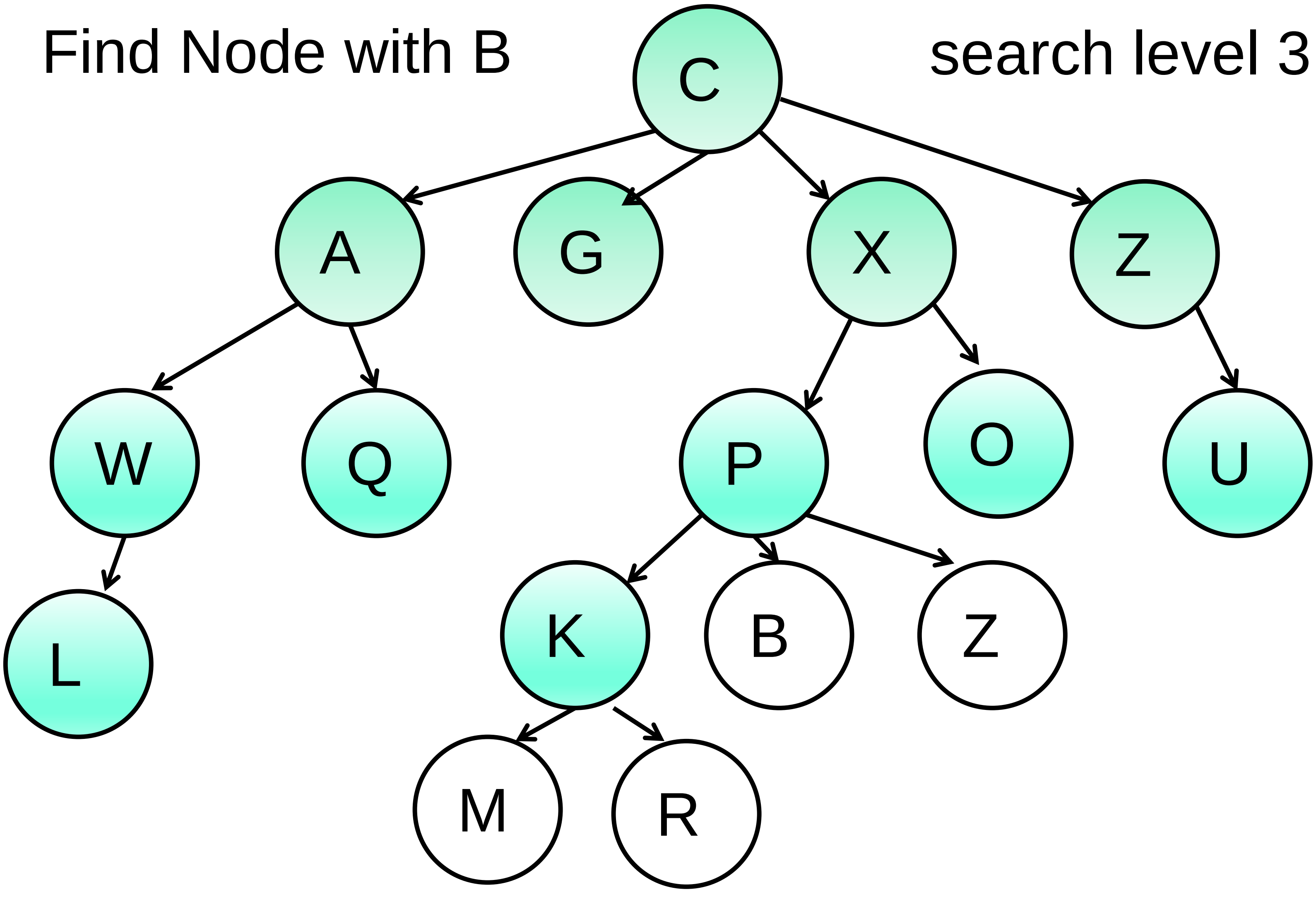
search level 3 next



Breadth First Search

Find Node with B

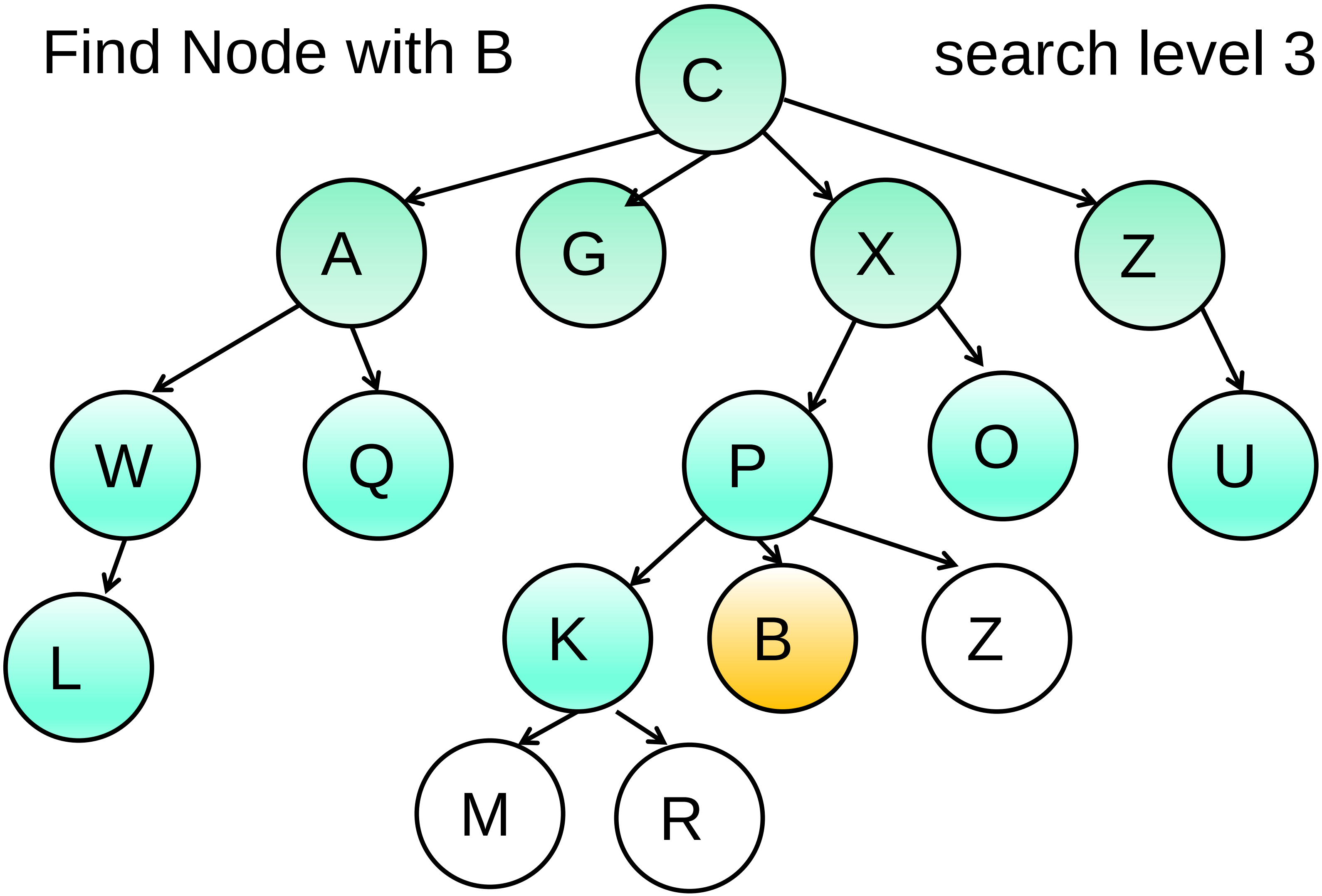
search level 3 next



Breadth First Search

Find Node with B

search level 3 next

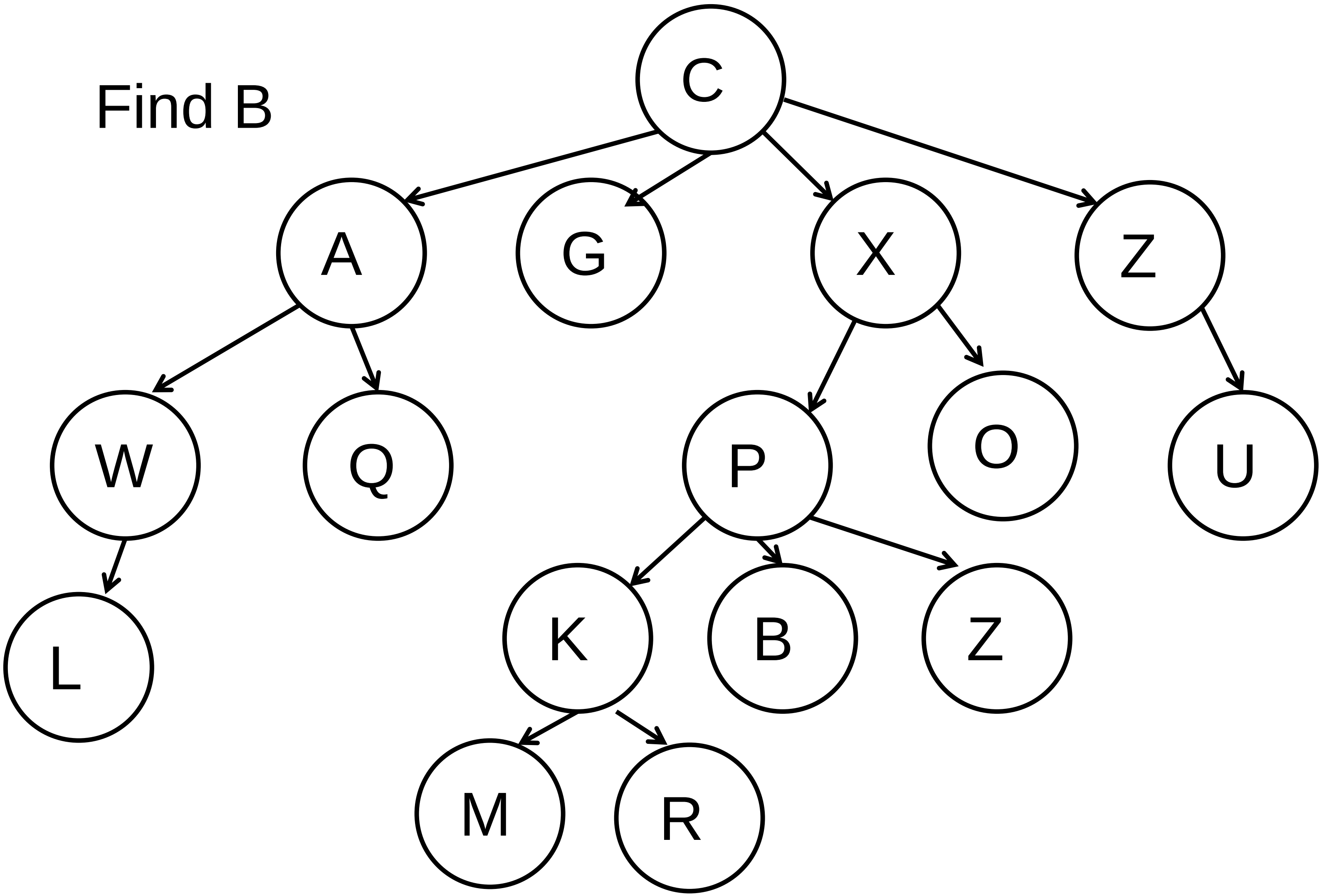


BFS - DFS

- Breadth first search typically implemented with a Queue
- Depth first search typically implemented with a stack, implicit with recursion or iteratively with an explicit stack
- which technique do I use?
  – depends on the problem

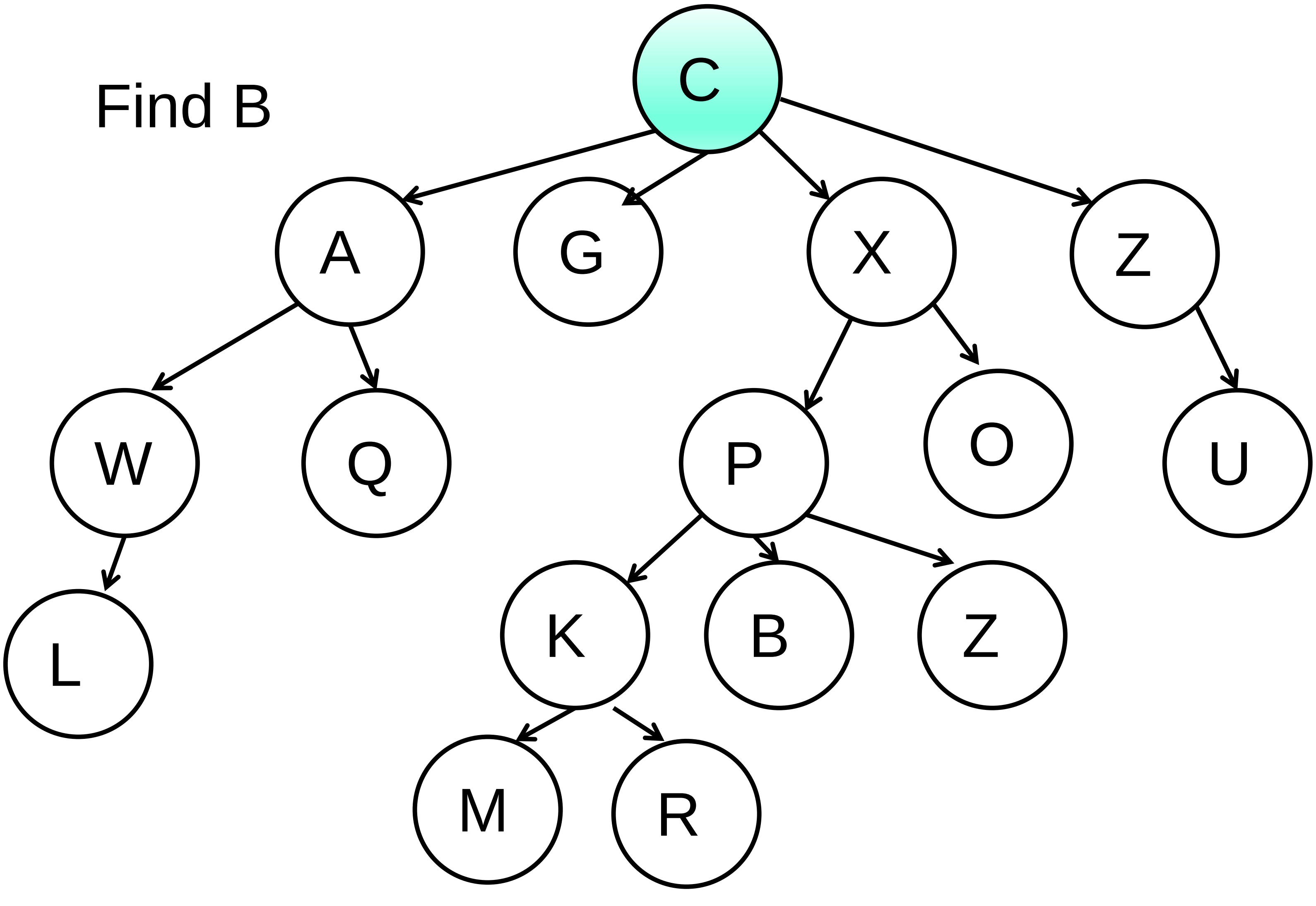
Depth First Search of Tree

Find B



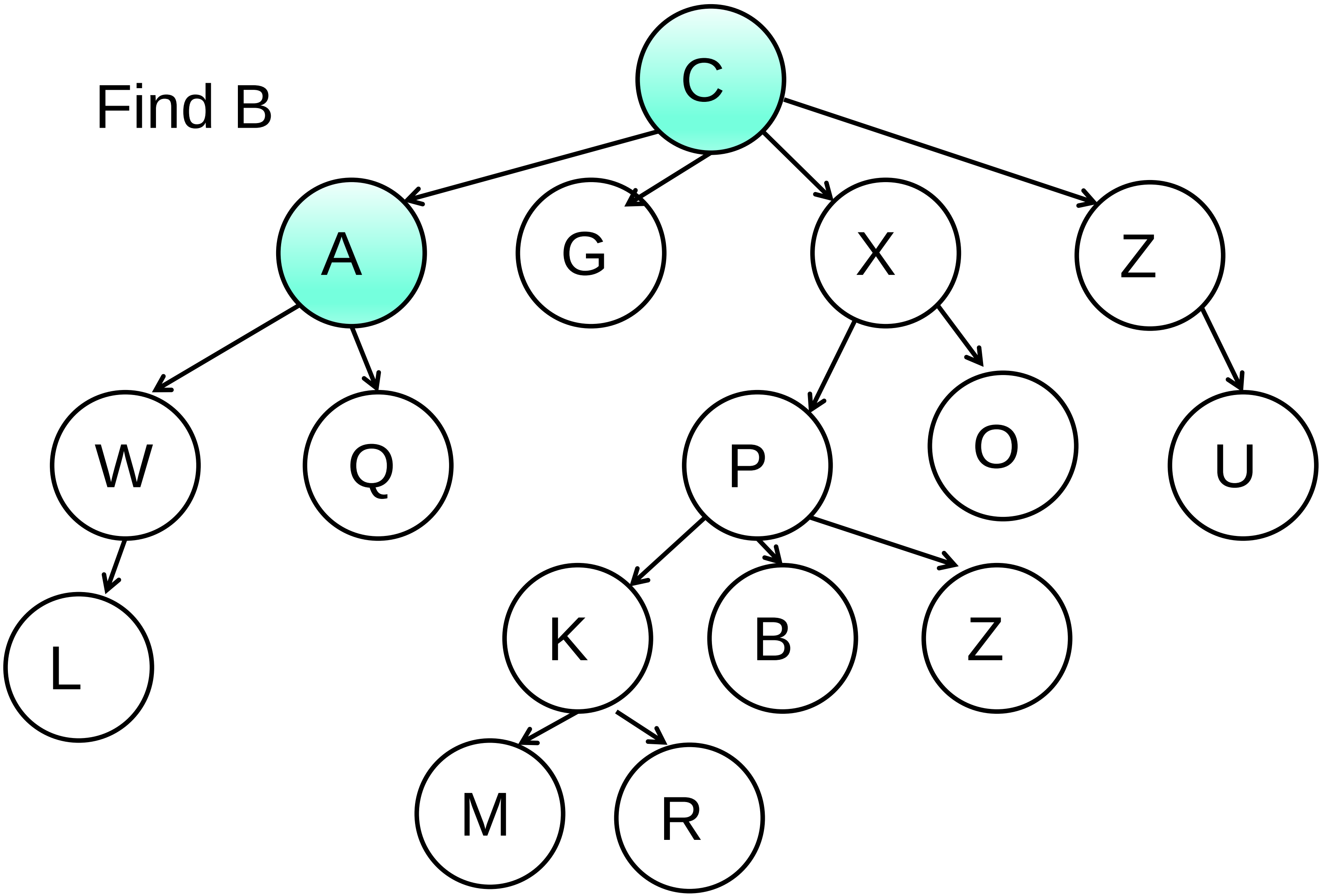
Depth First Search of Tree

Find B



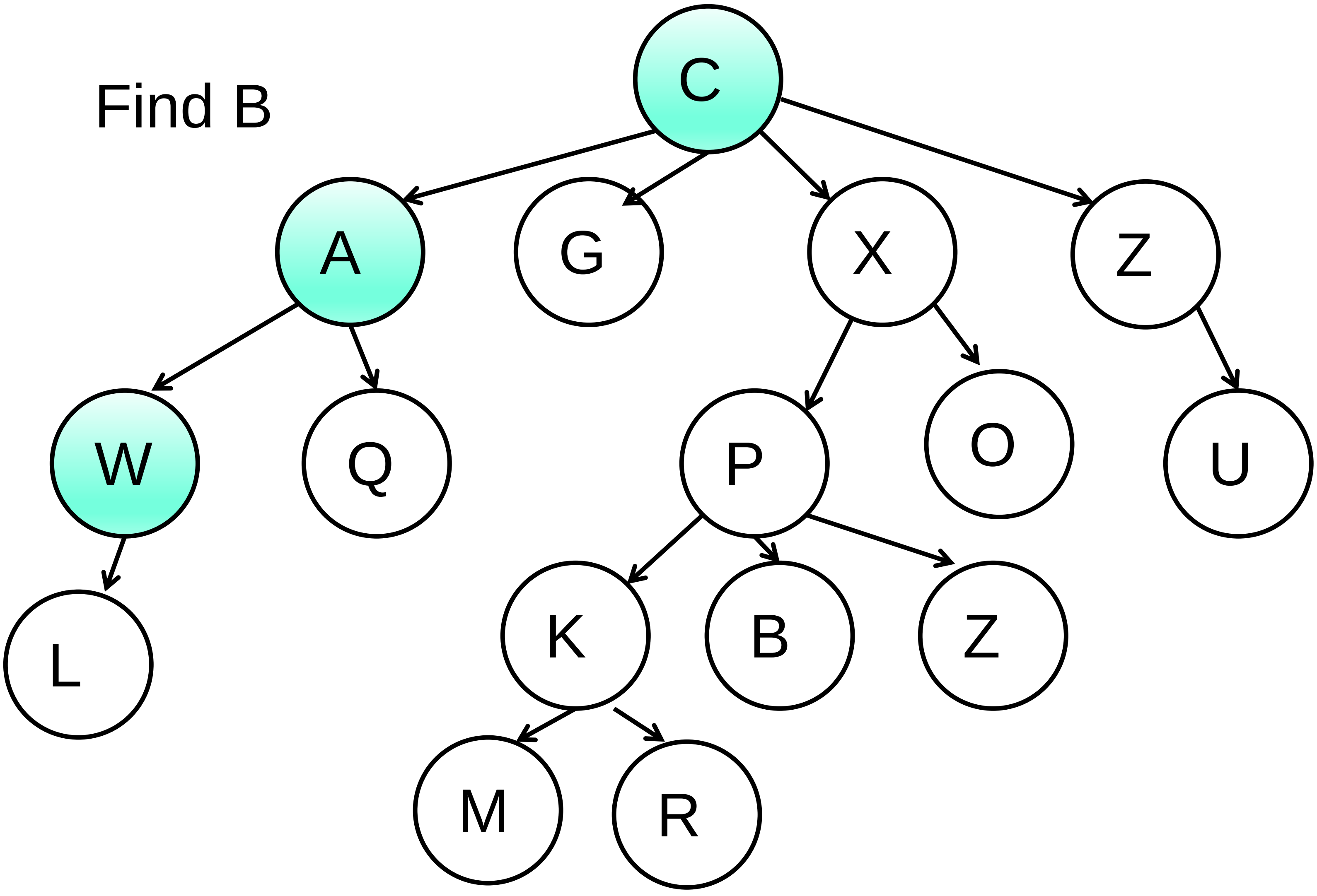
Depth First Search of Tree

Find B



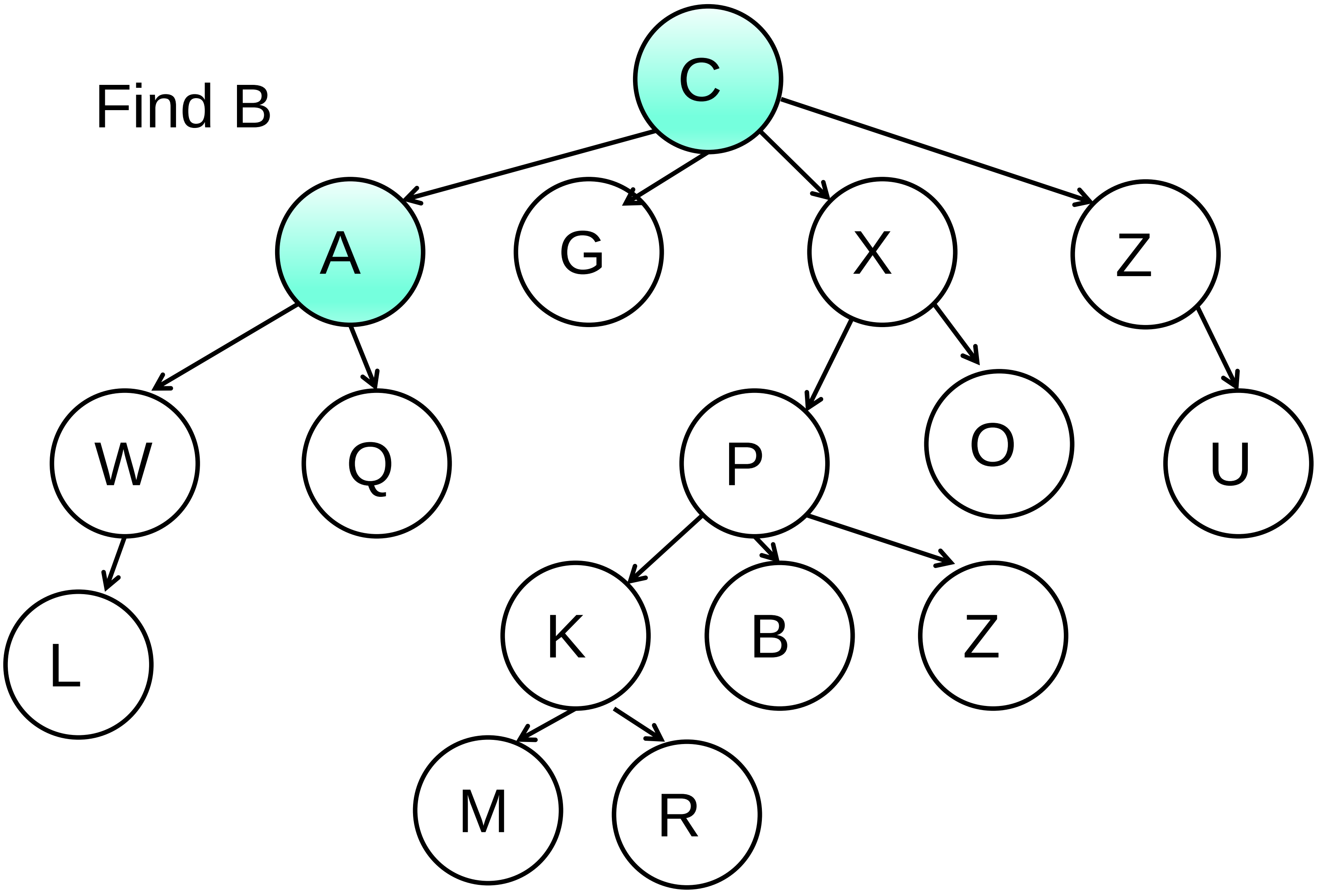
Depth First Search of Tree

Find B



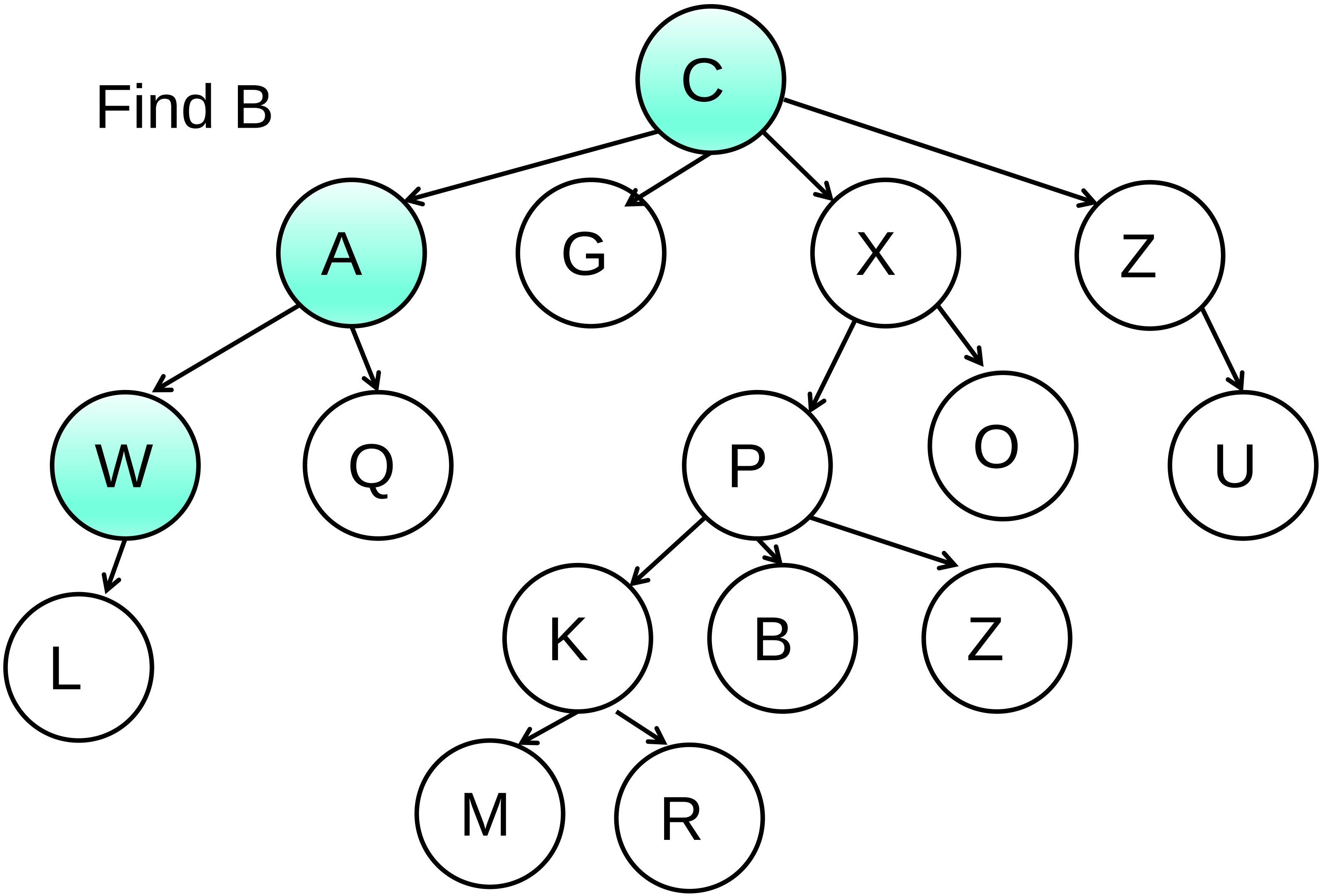
Depth First Search of Tree

Find B



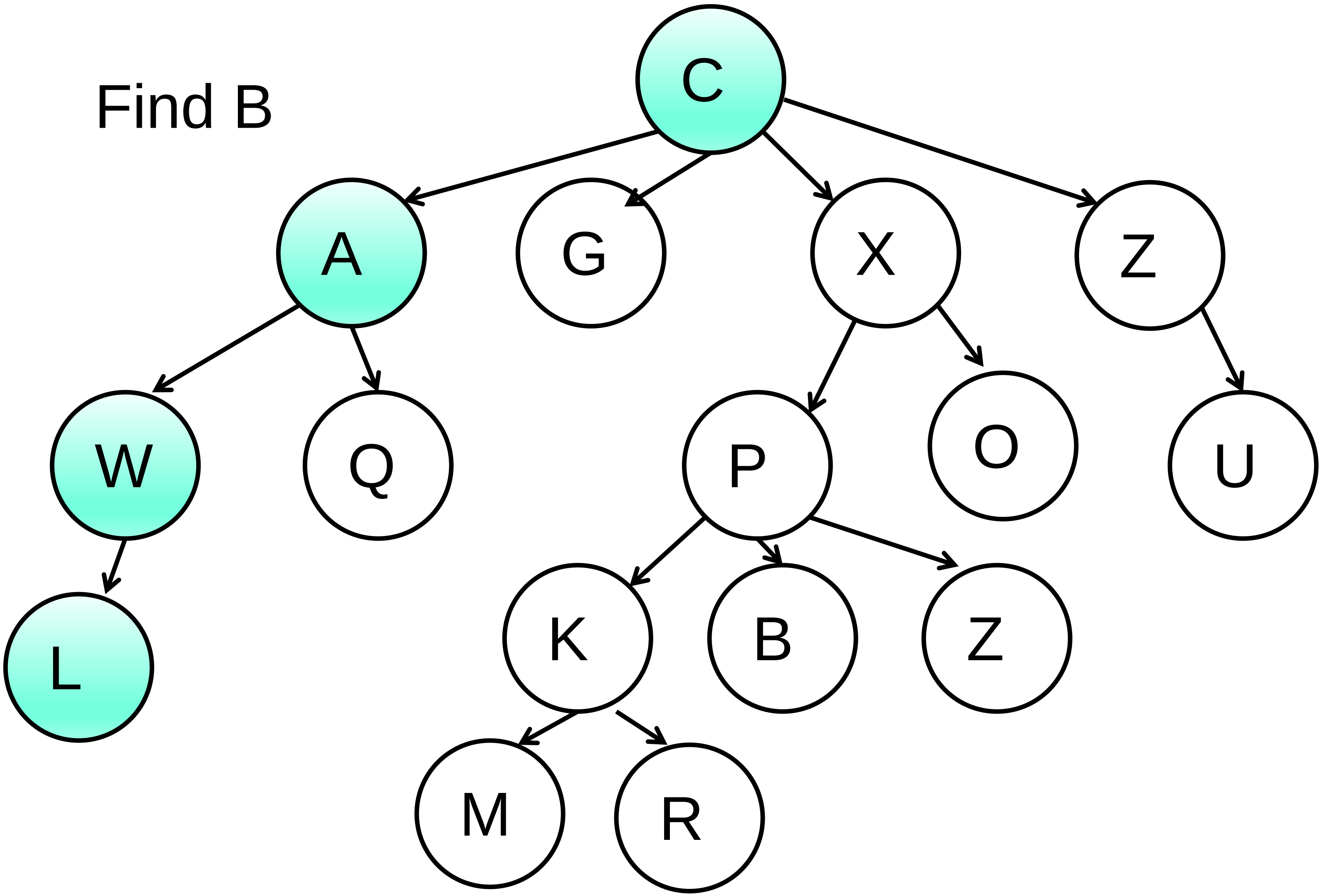
Depth First Search of Tree

Find B



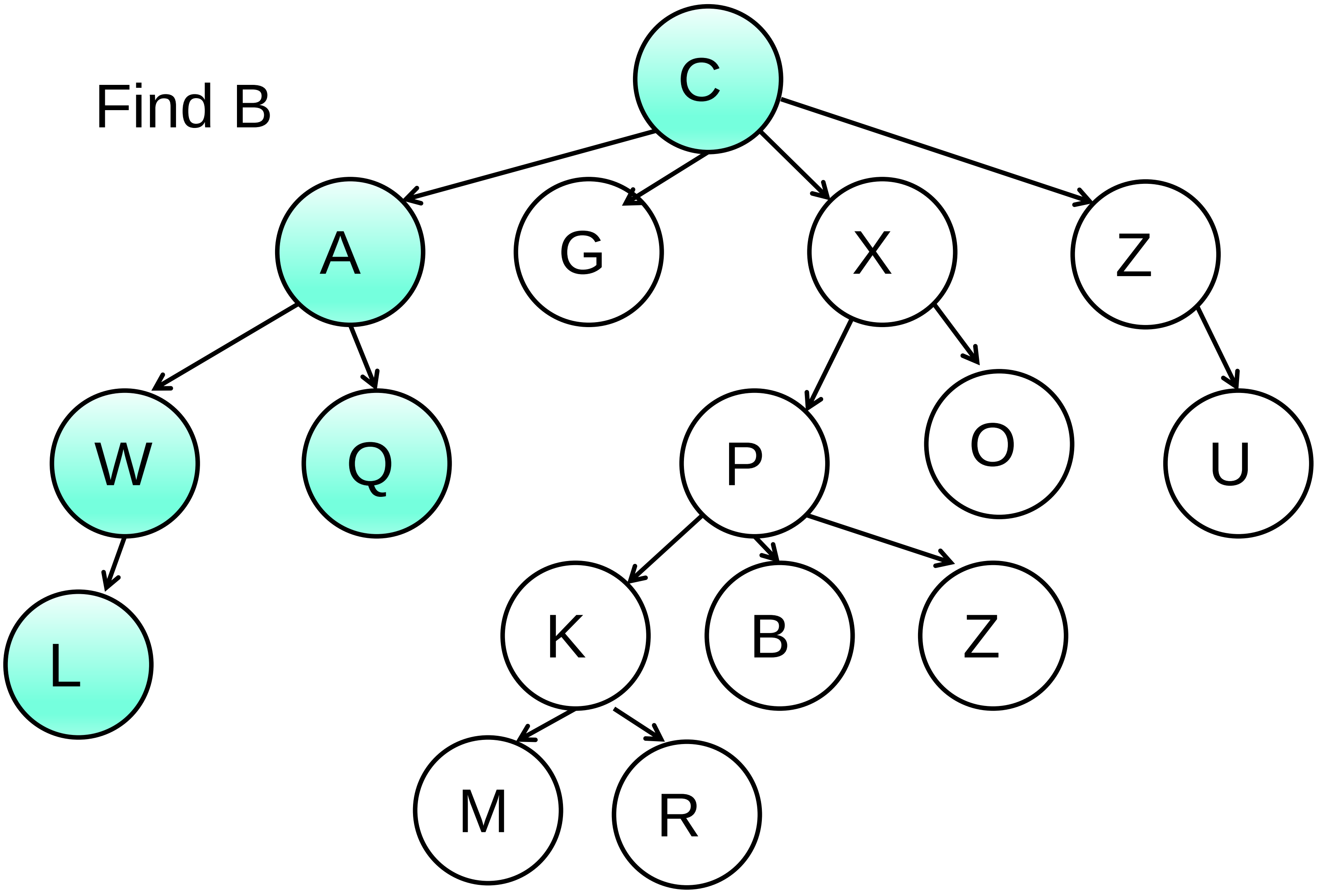
Depth First Search of Tree

Find B



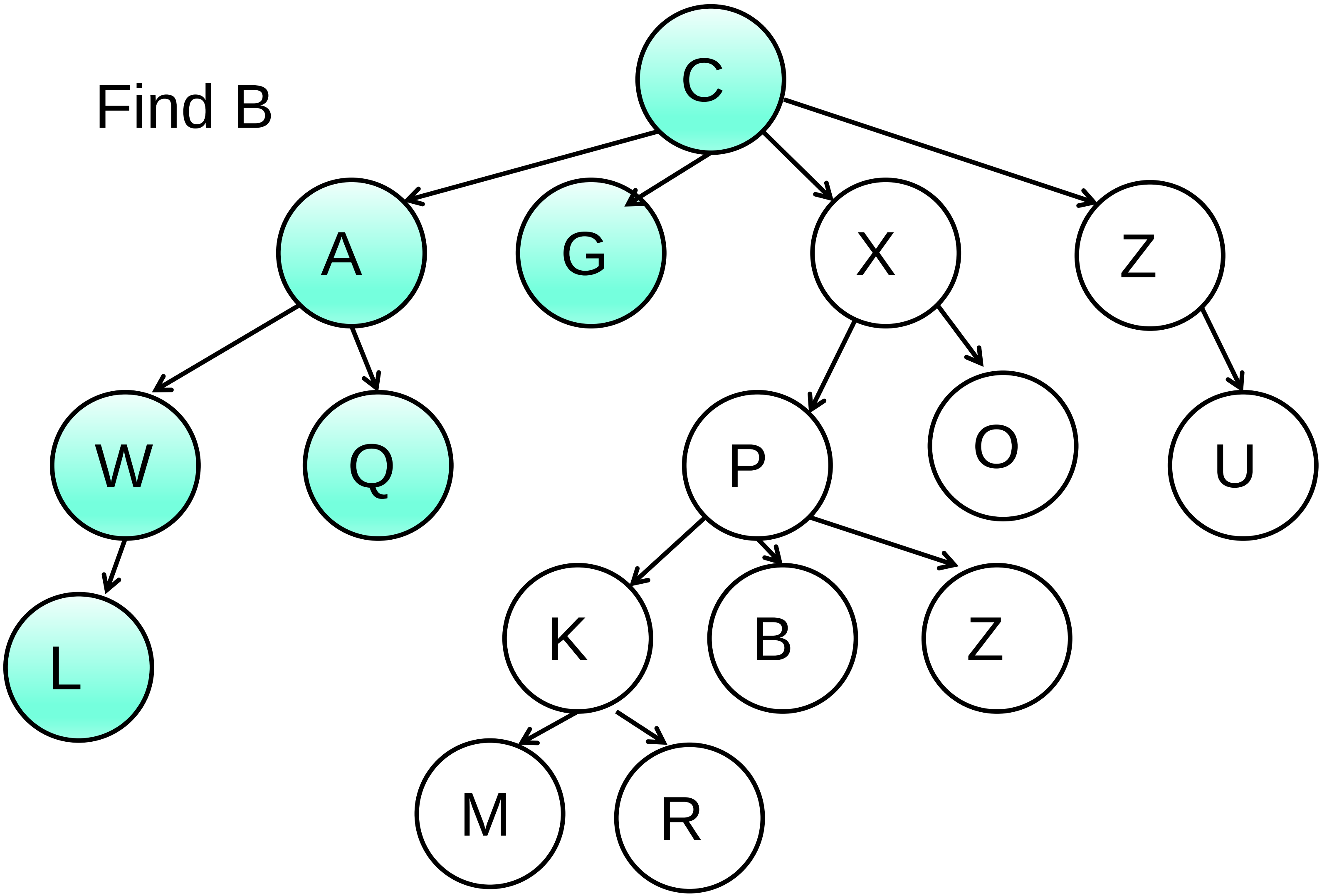
Depth First Search of Tree

Find B



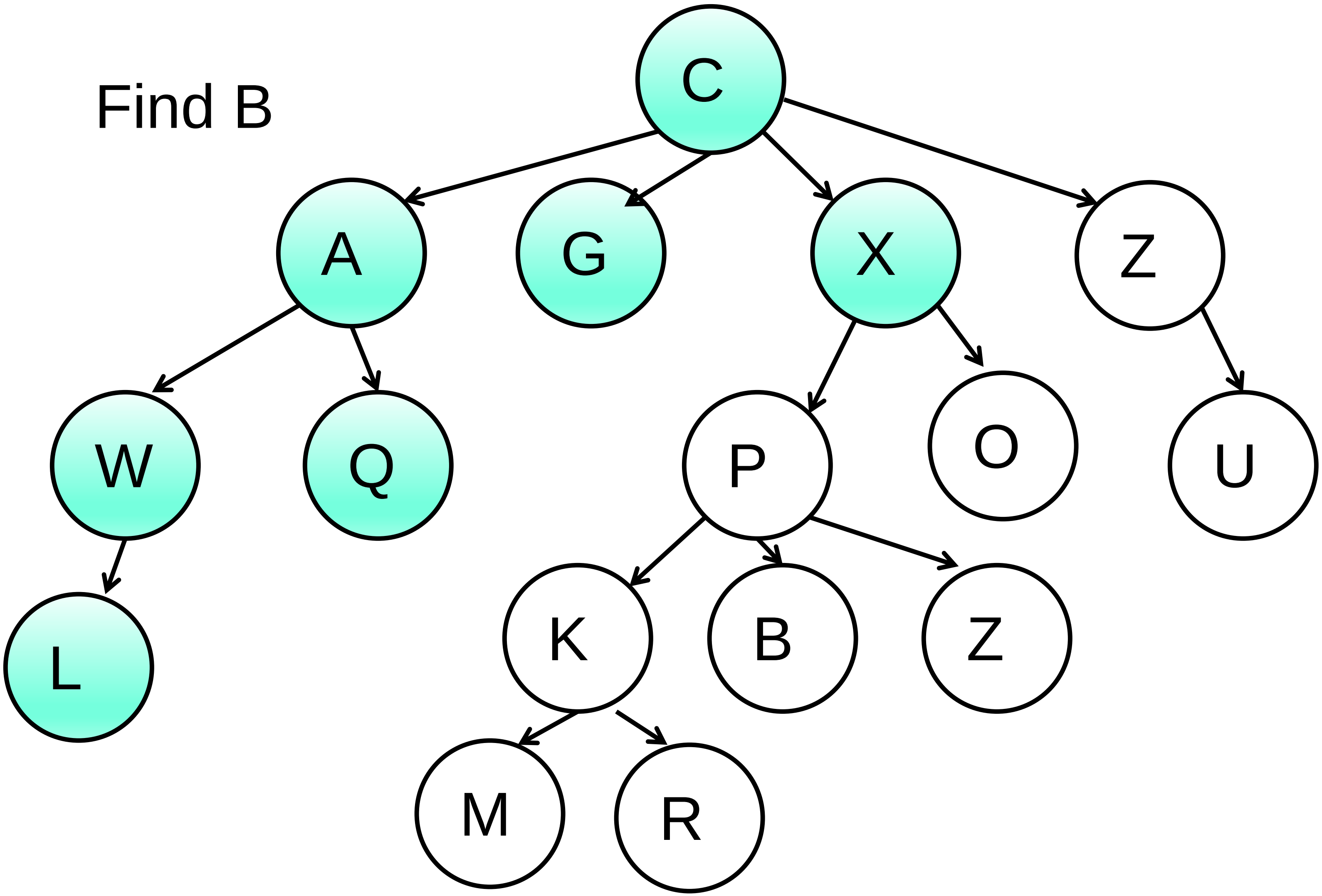
Depth First Search of Tree

Find B



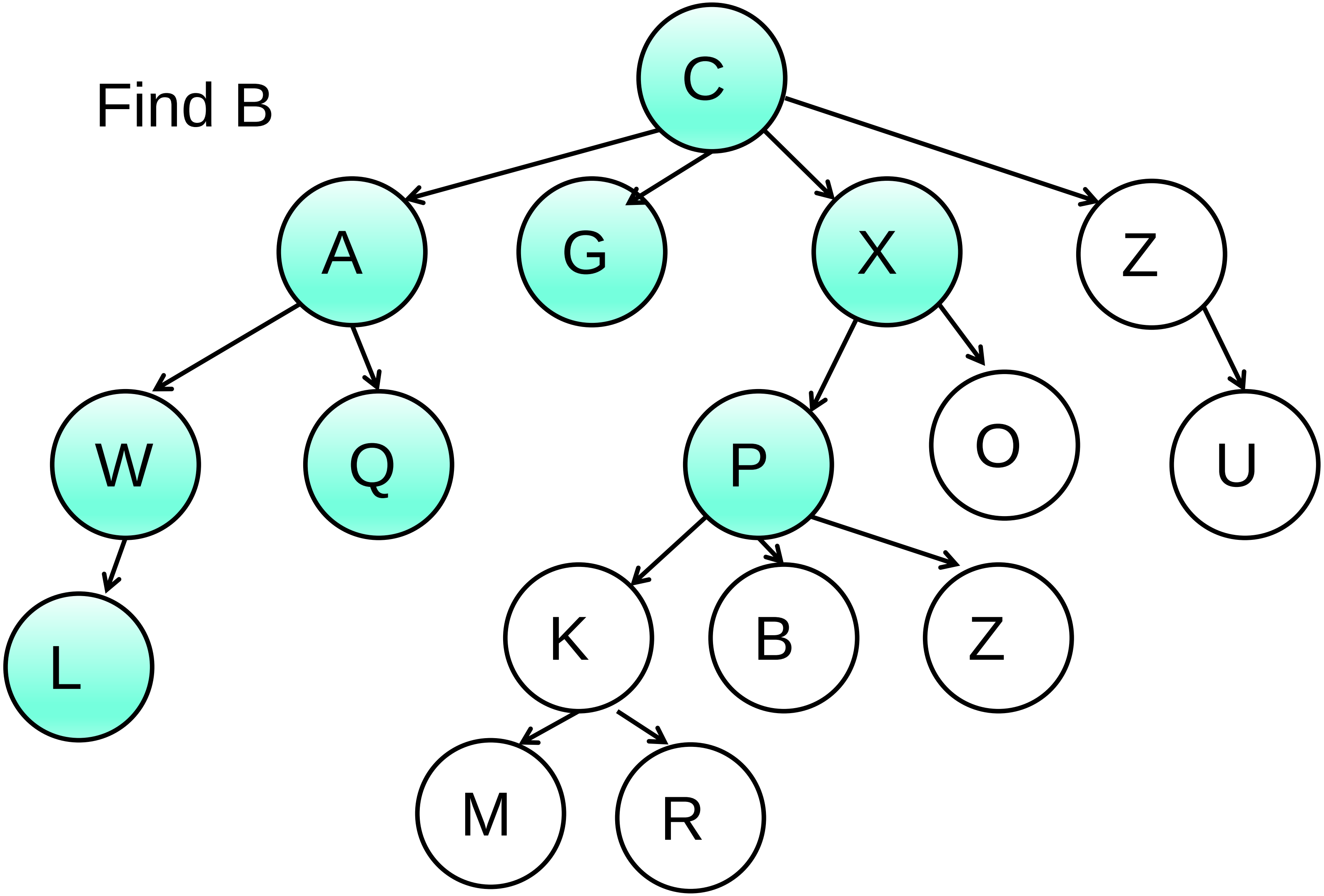
Depth First Search of Tree

Find B



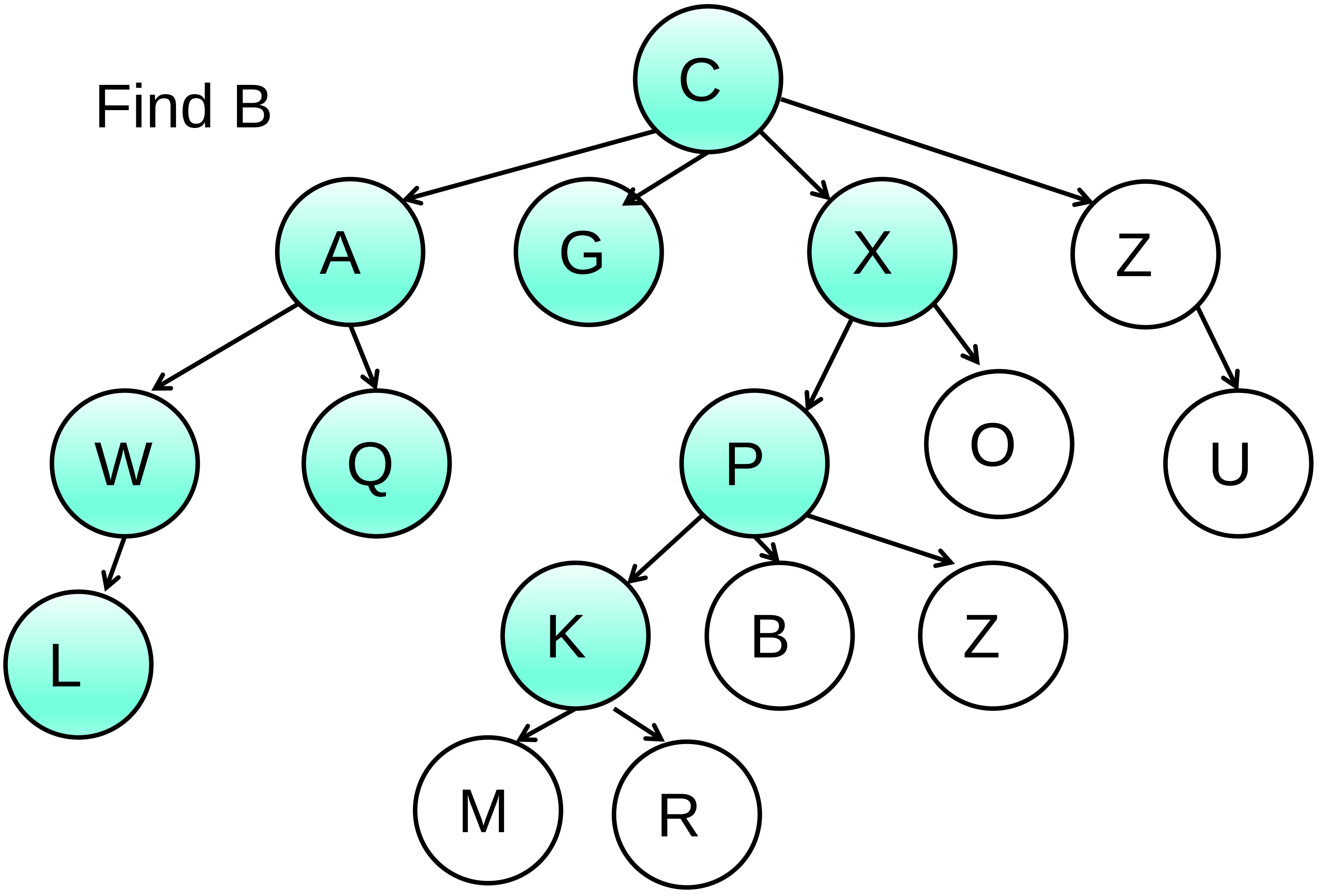
Depth First Search of Tree

Find B



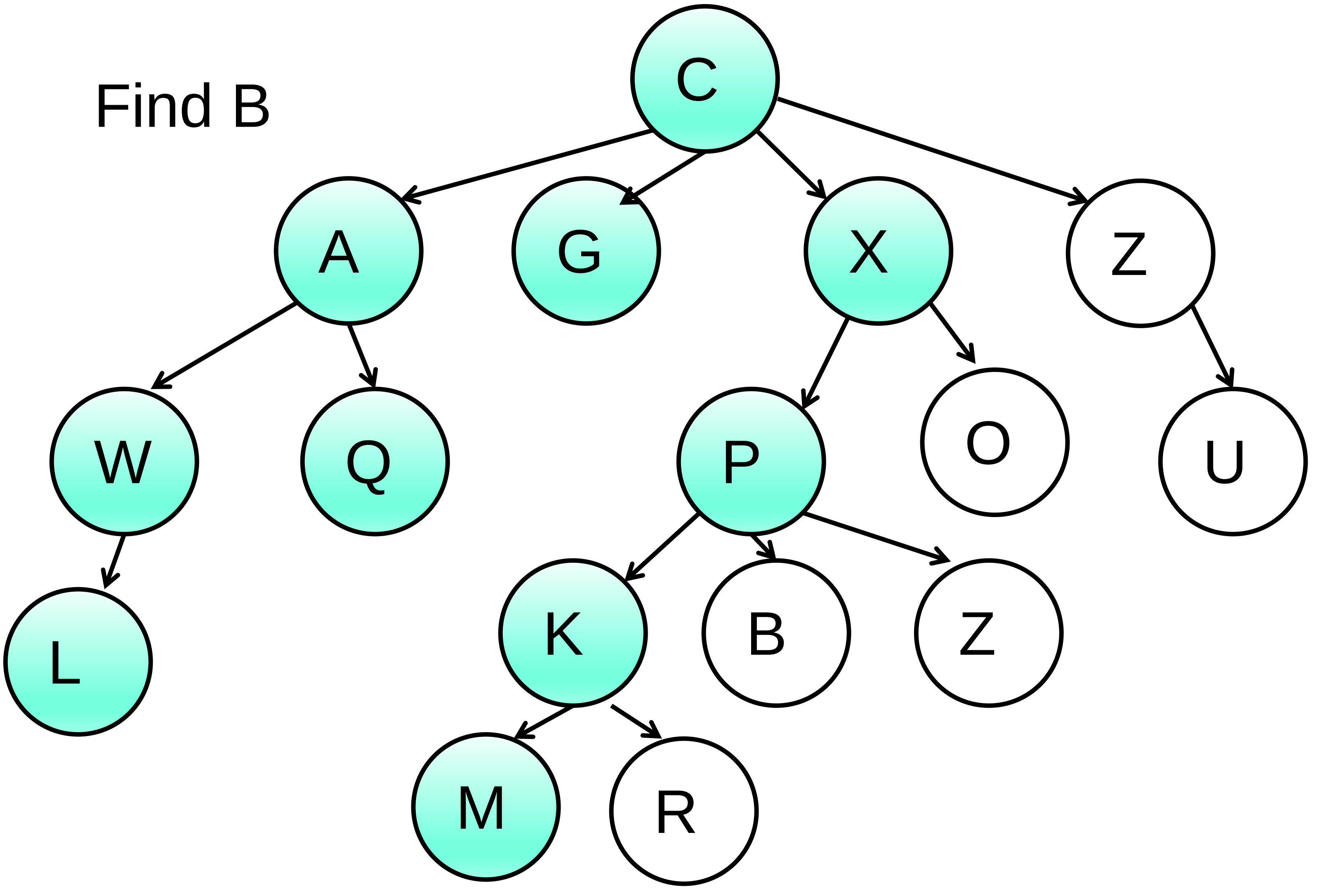
Depth First Search of Tree

Find B



Depth First Search of Tree

Find B



Depth First Search of Tree

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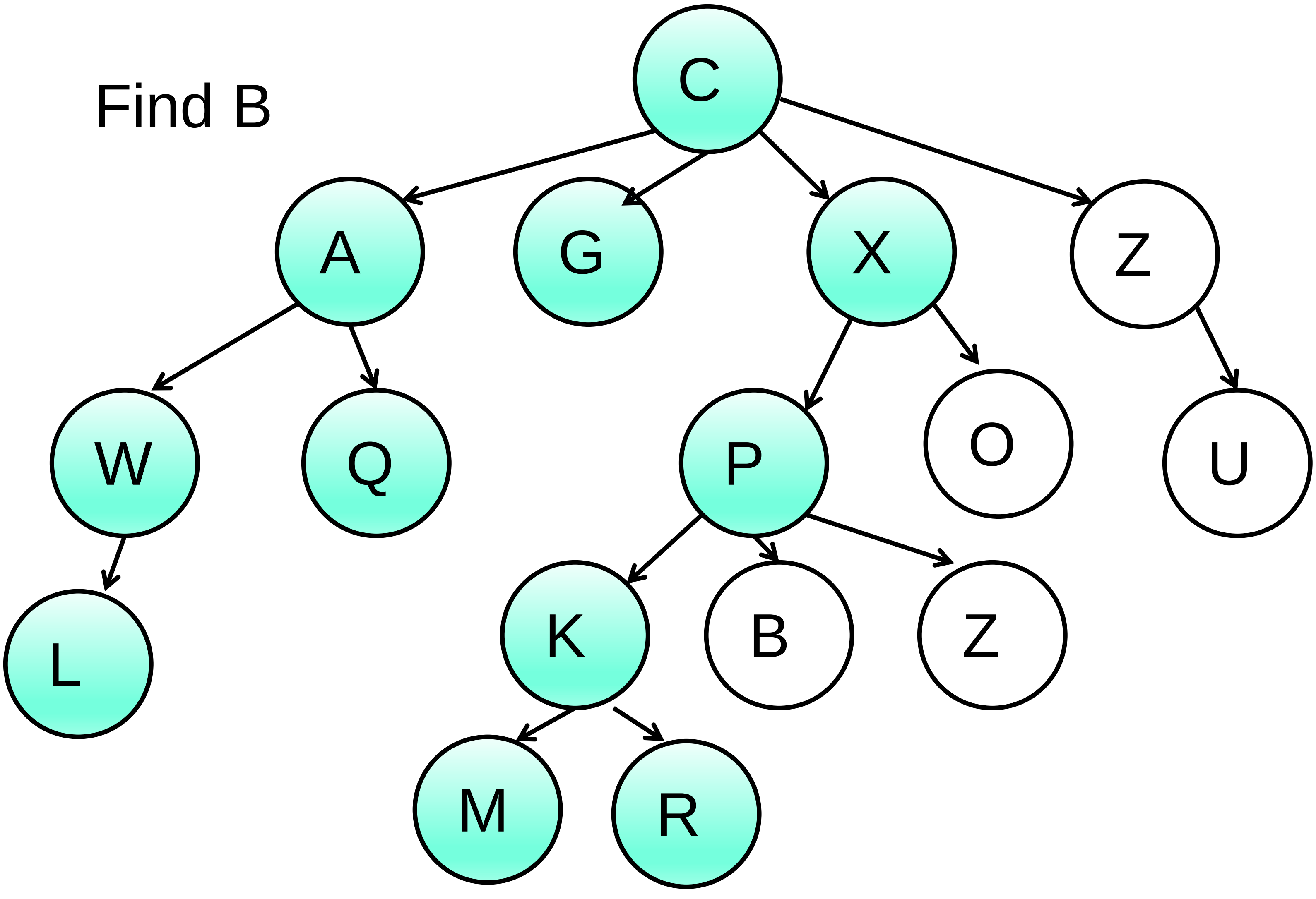

Depth First Search of Tree

Find B