"Yes. Shrubberies are my trade. I am a shrubber. My name is 'Roger the Shrubber'. I arrange, design, and sell shrubberies."

-Monty Python and The Holy Grail
The Problem with Linked Lists

- Accessing an item from a linked list takes $O(N)$ time for an arbitrary element.
- Binary trees can improve upon this and reduce access to $O(\log N)$ time for the average case.
- Expands on the binary search technique and allows insertions and deletions.
- Worst case degenerates to $O(N)$ but this can be avoided by using balanced trees (AVL, Red-Black).
Binary Search Trees

- A binary search tree is a binary tree in which every node's left subtree holds values less than the node's value, and every right subtree holds values greater than the node's value.

- A new node is added as a leaf.
BST Insertion

- Add the following values one at a time to an initially empty binary search tree using the naïve algorithm:

  90  20  9  98  10  28  -25

- What is the resulting tree?
Traversals

- What is the result of an inorder traversal of the resulting tree?
- How could a preorder traversal be useful?
Clicker Question 1

After adding N distinct elements in random order to a Binary Search Tree what is the expected height of the tree?

A. $O(N^{1/2})$
B. $O(\log N)$
C. $O(N)$
D. $O(N \log N)$
E. $O(N^2)$
Clicker Question 1

After adding $N$ distinct elements to a Binary Search Tree what is the worst case height of the tree?

A. $O(N^{1/2})$
B. $O(\log N)$
C. $O(N)$
D. $O(N\log N)$
E. $O(N^2)$
Node for Binary Search Trees

d public class BSTNode<E extends Comparable<E> {  
 private Comparable<E> myData;
 private BSTNode<E> myLeft;
 private BSTNode<E> myRightC;

d public BinaryNode(E item)  
 {   myData = item;   }

d public E getValue()  
 {   return myData;   }

d public BinaryNode<E> getLeft()  
 {   return myLeft;   }

d public BinaryNode<E> getRight()  
 {   return myRight;   }

d public void setLeft(BSTNode<E> b)  
 {   myLeft = b;   }  
 // setRight not shown
Worst Case Performance

- Insert the following values into an initially empty binary search tree using the traditional, naïve algorithm:

  2 3 5 7 11 13 17

- What is the height of the tree?
- What is the worst case height of a BST?
More on Implementation

- Many ways to implement BSTs
- Using nodes is just one and even then many options and choices

```java
public class BinarySearchTree<E extends Comparable<E>> {
    private BSTNode<E> root;
    private int size;
}
```
Add an Element, Recursive
Add an Element, Iterative
### Clicker Question 2

What are the best case and worst case order to add N distinct elements, one at a time, to an initially empty binary search tree?

<table>
<thead>
<tr>
<th>Best</th>
<th>Worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $O(N)$</td>
<td>$O(N)$</td>
</tr>
<tr>
<td>B. $O(N\log N)$</td>
<td>$O(N\log N)$</td>
</tr>
<tr>
<td>C. $O(N)$</td>
<td>$O(N\log N)$</td>
</tr>
<tr>
<td>D. $O(N\log N)$</td>
<td>$O(N^2)$</td>
</tr>
<tr>
<td>E. $O(N^2)$</td>
<td>$O(N^2)$</td>
</tr>
</tbody>
</table>

```java
// given int[] data
// no duplicates in data
BST<Integer> b = new BST<Integer>();
for (int x : data)
    b.add(x);
```
Performance of Binary Trees

- For the three core operations (add, access, remove) a binary search tree (BST) has an average case performance of $O(\log N)$
- Even when using the *naïve insertion / removal algorithms*
  - no checks to maintain balance
  - balance achieved based on the randomness of the data inserted
Remove an Element

- Three cases
  - node is a leaf, 0 children (easy)
  - node has 1 child (easy)
  - node has 2 children (interesting)
Properties of a BST

- The minimum value is in the left most node
- The maximum value is in the right most node
  - useful when removing an element from the BST
Alternate Implementation

- In class examples of dynamic data structures have relied on *null terminated ends*.
  - Use null to show end of list, no children
- Alternative form
  - use structural recursion and polymorphism
public interface BST {
    public int size();
    public boolean contains(Comparable obj);
    public boolean add(Comparable obj);
}
public class EmptyBST implements BST {

    private static EmptyBST theOne = new EmptyBST();

    private EmptyBST(){}

    public static EmptyBST getEmptyBST(){ return theOne; }

    public NRBST add(Comparable obj) { return new NRBST(obj); }

    public boolean contains(Comparable obj) { return false; }

    public int size() { return 0; }
}
public class NEBST implements BST {

    private Comparable data;
    private BST left;
    private BST right;

    public NEBST(Comparable d) {
        data = d;
        right = EmptyBST.getEmptyBST();
        left = EmptyBST.getEmptyBST();
    }

    public BST add(Comparable obj) {
        int val = obj.compareTo(data);
        if (val < 0)
            left = left.add(obj);
        else if (val > 0)
            right = right.add(obj);
        return this;
    }
}
public boolean contains(Comparable obj) {
    int val = obj.compareTo(data);
    if (val == 0)
        return true;
    else if (val < 0)
        return left.contains(obj);
    else
        return right.contains(obj);
}

public int size() {
    return 1 + left.size() + right.size();
}