"Yes. Shrubberies are my trade. I am a shrubber. My name is 'Roger the Shrubber'. I arrange, design, and sell shrubberies."

-Monty Python and The Holy Grail

The Problem with Linked Lists

- Accessing a item from a linked list takes O(N) time for an arbitrary element
- Binary trees can improve upon this and reduce access to O(log N) time for the average case
- Expands on the binary search technique and allows insertions and deletions
- Worst case degenerates to O(N) but this can be avoided by using balanced trees (AVL, Red-Black)

Binary Search Trees

- A binary search tree is a binary tree in which every node's left subtree holds values less than the node's value, and every right subtree holds values greater than the node's value.
- A new node is added as a leaf.

BST Insertion

- Add the following values one at a time to an initially empty binary search tree using the traditional, naïve algorithm:
  
  90  20  9  98  10  28  -25

- What is the resulting tree?
Traversals

- What is the result of an inorder traversal of the resulting tree?
- How could a preorder traversal be useful?

Clicker Question 1

- After adding N distinct elements in random order to a Binary Search Tree what is the expected height of the tree?

A. \( O(N^{1/2}) \)
B. \( O(\log N) \)
C. \( O(N) \)
D. \( O(N\log N) \)
E. \( O(N^2) \)

Node for Binary Search Trees

```java
public class BSTNode<E extends Comparable<E> {  
    private Comparable<E> myData;
    private BSTNode<E> myLeft;
    private BSTNode<E> myRight;

    public BSTNode(E item)  
    {  
        myData = item;
    }

    public E getValue()  
    {  
        return myData;
    }

    public BSTNode<E> getLeft()  
    {  
        return myLeft;
    }

    public BSTNode<E> getRight()  
    {  
        return myRight;
    }

    public void setLeft(BSTNode<E> b)  
    {  
        myLeft = b;
    }  
    // setRight not shown
}
```

Worst Case Performance

- Insert the following values into an initially empty binary search tree using the traditional, naïve algorithm:

\[ 2 \ 3 \ 5 \ 7 \ 11 \ 13 \ 17 \]

- What is the height of the tree?
- What is the worst case height of a BST?
More on Implementation

- Many ways to implement BSTs
- Using nodes is just one and even then many options and choices

```java
public class BinarySearchTree<E extends Comparable<E>> { 
    private BSTNode<E> root;
    private int size;
}
```

Add an Element, Recursive

Add an Element, Iterative

Clicker Question 2

- What are the best case and worst case order to add N distinct elements, one at a time, to an initially empty binary search tree?

<table>
<thead>
<tr>
<th></th>
<th>Best</th>
<th>Worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>O(N)</td>
<td>O(N)</td>
</tr>
<tr>
<td>B</td>
<td>O(NlogN)</td>
<td>O(NlogN)</td>
</tr>
<tr>
<td>C</td>
<td>O(N)</td>
<td>O(NlogN)</td>
</tr>
<tr>
<td>D</td>
<td>O(NlogN)</td>
<td>O(N^2)</td>
</tr>
<tr>
<td>E</td>
<td>O(N^2)</td>
<td>O(N^2)</td>
</tr>
</tbody>
</table>

```
// given int[] data
// no duplicates in data
BST<Integer> b = new BST<Integer>();
for(int x : data) 
    b.add(x);
```
Performance of Binary Trees

- For the three core operations (add, access, remove) a binary search tree (BST) has an average case performance of $O(\log N)$
- Even when using the naïve insertion / removal algorithms
  - no checks to maintain balance
  - balance achieved based on the randomness of the data inserted

Remove an Element

- Three cases
  - node is a leaf, 0 children (easy)
  - node has 1 child (easy)
  - node has 2 children (interesting)

Properties of a BST

- The minimum value is in the left most node
- The maximum value is in the right most node
  - useful when removing an element from the BST

Alternate Implementation

- In class examples of dynamic data structures have relied on null terminated ends.
  - Use null to show end of list, no children
- Alternative form
  - use structural recursion and polymorphism
### BST Interface

```java
public interface BST {
    public int size();
    public boolean contains(Comparable obj);
    public boolean add(Comparable obj);
}
```

### EmptyBST

```java
public class EmptyBST implements BST {
    private static EmptyBST theOne = new EmptyBST();
    private EmptyBST(){}
    public static EmptyBST getEmptyBST(){ return theOne; }
    public NEBST add(Comparable obj) { return new NEBST(obj); }
    public boolean contains(Comparable obj) { return false; }
    public int size() { return 0; }
}
```

### Non Empty BST – Part 1

```java
public class NEBST implements BST {
    private Comparable data;
    private BST left;
    private BST right;

    public NEBST(Comparable d){
        data = d;
        right = EmptyBST.getEmptyBST();
        left = EmptyBST.getEmptyBST();
    }

    public BST add(Comparable obj) {
        int val = obj.compareTo( data );
        if( val < 0 )
            left = left.add( obj );
        else if( val > 0 )
            right = right.add( obj );
        return this;
    }
}
```

### Non Empty BST – Part 2

```java
public boolean contains(Comparable obj){
    int val = obj.compareTo(data);
    if( val == 0 )
        return true;
    else if( val < 0)
        return left.contains(obj);
    else
        return right.contains(obj);
}

public int size() {
    return 1 + left.size() + right.size();
}
```