336, Fall 2009, Professor Warnow
Homework #1

Basic information:

- Due date: September 8, 2009 (in class), by 2:15.
- You can discuss the problems with other students, but you should write it up yourself. Also, later quizzes are likely to be based upon these (and other homeworks), so please be sure you understand the solutions you write down.
- Please put your name (clearly) on each page of your homework, and staple all the pages together.

Problems:

1. List all mathematics and computer science courses you have taken at the University of Texas, the semester you took the course, and professor who taught the course.
2. Give the set-theoretic notation for all positive rational numbers.
3. Give the set-theoretic notation for all positive irrational numbers.
4. Give the set-theoretic notation for all positive real numbers that are not multiples of 3.
5. Give the set-theoretic notation for all functions from the reals to the integers.
6. Express using mathematical notation that a function f from the set A to the set B is $1-1$.
7. Express using mathematical notation that a function f from the set A to the set B is onto.
8. Write out the truth tables for the following statements:
   - $(A \Rightarrow B) \wedge (\neg A)$
   - $A \wedge (B \Rightarrow \neg A) \wedge \neg B$
   - $A \wedge (\neg B \Rightarrow \neg A) \wedge \neg B$
9. Consider the following statements:
   - $\forall x \in S \; \exists y \in S \; s.t. \; x < y$
   - $\exists x \in S \; s.t. \; \forall y \in S - \{x\}, x < y$

(a) For each of these statements, determine whether the statement is true for each of the following ways of defining S:

- $S = \{1, 2, 3, ...\}$
- $S = \{-1, -2, -3, ...\}$
- $S = \{2, 4, 6, 8, 10\}$
- $S = \{x \in R : -3 \leq x \leq 3\}$
- $S = \{x \in R : -3 < x < 3\}$
- $S = \emptyset$

(b) Write out the negation of each of these statements.

10. Express the following problem as a graph-theoretic problem:

    - You want to divide up all the students at the University of Texas into groups, so that no group has any two students who dislike each other. You wish to minimize the total number of groups. You can assume that "dislike" is symmetric.

    What are the vertices, and how are the edges defined? What is the output? (Do not try to design an algorithm for this problem!)

11. Draw all simple graphs on four unlabelled vertices. How many of them are connected?

12. Draw all rooted binary trees with four labelled leaves a, b, c, d.

13. Draw all rooted trees with four labelled leaves a, b, c, d.

14. Draw all unrooted binary trees with four labelled leaves a, b, c, d.

15. Draw all unrooted trees with four labelled leaves a, b, c, d.

16. Write down all functions from $\{1, 2\}$ to $\{1, 2\}$. How many of these functions are $1 - 1$? How many are onto?

17. Prove by induction on n that $1 + 2 + \ldots + n = n(n + 1)/2$.