

Quiz 5, CS 336, Tandy Warnow

Give your solution for each problem. No explanation needed.

1. Consider a set  $X$  of  $N$  people, where  $X$  includes Henry, Sally, and Bob (so  $N \geq 3$ ). Express your answers as a function of  $N$ .
  - (a) How many subsets of  $X$  are there?
  - (b) How many subsets of  $X$  have exactly three people?
  - (c) How many subsets of  $X$  have at least three people?
  - (d) How many subsets of  $X$  have both Henry and Sally?
  - (e) How many subsets of  $X$  have neither Henry nor Sally?
  - (f) How many subsets of  $X$  have at most one of Henry or Sally?
  - (g) How many ways can you select three people from  $X$ , and put them in a row?
2. Prove by induction on  $n$  that  $F(n) \geq n$  for all  $n \geq 1$  where  $F(n)$  is defined by  $F(1) = 1, F(2) = 2$  and  $F(n) = F(n - 2) + 3$  when  $n \geq 3$ .
3. Consider a sequence of sets  $A_1, A_2, A_3, \dots$  where  $A_1 = \{1\}$  and  $A_n = A_{n-1} \cup \{2n, 2n - 1\}$  where  $n \geq 2$ .
  - (a) What is  $A_2$ ?
  - (b) What is  $A_3$ ?
  - (c) Prove, by induction on  $n$ , that  $|A_n| = 2n - 1$ .