

CS 336, Professor Warnow
Homework #5

Due: October 6, 2009, by 2:15 PM

1. Consider the following problem: *Heaviest Consecutive Subsequence*
 - Input: array $A[1..n]$ of integers
 - Output: indices i, j with $i \leq j$ such that $\sum_{k=i}^j A_k$ is maximized.
 - (a) What is the heaviest consecutive subsequence for the following input:
 $1, -3, 5, -4, 8, 6, -10, 1, 4$?
 - (b) Design an $O(n)$ dynamic programming algorithm to compute the *weight* of the heaviest consecutive subsequence problem by computing solutions to subproblems $M[i]$, where $i = 1, 2, \dots, n$, and $M[i]$ is defined to be the weight of the heaviest consecutive subsequence of $A[1..i]$ that includes $A[i]$. *Give the algorithm in full detail.* (Maximum 50% credit if your algorithm is $O(n^2)$ instead of $O(n)$. Maximum 25% credit if your algorithm is $O(n^3)$.)
2. Prove by contradiction that $\sqrt{3}$ is not rational.
3. Find a closed form solution for $F(n) = 1 + 3 + 5 + \dots + (2n - 1)$, and prove it correct by induction on n .
4. Let $A = \{-1, -2, -3, -4, -5, 0, 1, 2\}$. Compute the following:
 - (a) $\{x \in A : -x \in A\}$
 - (b) $\{x \in A : x + 1 \in A\}$
 - (c) $\{X \subseteq A : \text{if } x \in X \text{ then } -x \in X\}$
 - (d) $\{X \subseteq A : \text{if } [x \in X \text{ and } y \in A \text{ and } y < x] \text{ then } y \in X\}$
 - (e) $\{X \subseteq A : \text{if } [x \in X \text{ and } y \in A \text{ and } x < y] \text{ then } y \in X\}$
 - (f) $\{X \subseteq A : \text{if } [x \in X \text{ and } y \in A \text{ and } x \neq y] \text{ then } y \in X\}$