

Homework #12, CS 336

Show all your work for these problems. Due November 24, 2009, at 2:05.

1. Let  $F(n)$  be defined for  $n \geq 1$ , by
  - $F(1) = 1$ ,
  - $F(2) = 5$ , and
  - $F(n) = F(n - 1) + F(n - 2)$  if  $n > 2$ .
  - (a) Give  $F(1)$ ,  $F(2)$ ,  $F(3)$ ,  $F(4)$ , and  $F(5)$ .
  - (b) Prove using strong induction that  $F(n) \geq 2^{n/2}$  for all  $n \geq 2$ .
2. Consider the following pseudo-code program for computing  $n!$  ( $n$  factorial), where  $n \geq 1$  is the input.

```
FACT := 1
i := 1
While (i < n) DO
    i := i + 1
    FACT := FACT * i
Return(FACT)
```

Prove that if the While loop terminates, that  $FACT = n!$  as desired.

3. Consider strings from the alphabet  $A, B, C, D \dots Z$ .
  - How many strings of length  $k$  are there?
  - How many strings of length 3 are there where they are all the same letter?
  - How many strings of length 3 are there where at least two letters are the same?
  - How many strings of length 3 are there that satisfy at least one of the following constraints:
    - Begin with A
    - Have B in the middle position
    - Have C in the third position