

Combinatorial counting problems

For each problem, give a solution and also an explanation for how you derived your solution. (Half points for the correct formula, and half points for a good explanation.)

1. Consider a set X of N men and N women, with $N \geq 1$. Express your answers as a function of N .
 - (a) How many subsets of X are there?
 - (b) How many non-empty subsets of X are there?
 - (c) How many subsets of X are there that are entirely men or entirely women?
 - (d) How many subsets of X are there that have at least one man and one woman?
 - (e) What proportion of the subsets are of mixed gender?
 - (f) For what values of N is the proportion of subsets that are mixed gender greater than $1/2$?
 - (g) Suppose you identify one particular person in the set X . How many subsets are there that do not include that person?
 - (h) Suppose you identify two people in the set X . How many subsets are there that include them both?
 - (i) How many ways can you line the people up in a row?
 - (j) How many ways can you line the people up in a row with all the men coming before all the women?
 - (k) How many ways can you line the people up in a row so that the men and women alternate, and a man begins the line?
 - (l) How many subsets can you create of exactly N people from X ?
2. Consider the particular set X with two men and two women, i.e. $X = \{Mary, Sally, Henry, Jim\}$.
 - (a) List all the subsets of X .
 - (b) List the non-empty subsets of X .
 - (c) List the subsets of X that are entirely men or entirely women.
 - (d) List the subsets of X that have at least one man and one woman.
 - (e) What proportion of the subsets of X are of mixed gender?
 - (f) How many subsets are there that do not include Jim?
 - (g) How many subsets are there that include both Jim and Henry?
 - (h) How many ways can you line the people up in a row?
 - (i) How many ways can you line the people up in a row with all the men coming before all the women?

- (j) How many ways can you line the people up in a row so that the men and women alternate, and a man begins the line?
- (k) How many subsets can you create of exactly 2 people from X ?