

### Homework #6: Solutions

1. Consider a set  $X$  of  $N$  men and  $N$  women, with  $N \geq 1$ . Express each answer as a function of  $N$ .
  - (a) There are  $2^{2N}$  subsets. (For  $N = 2$ , the answer is 16.)
  - (b) There are  $2^{2N} - 1$  subsets that are non-empty. (For  $N = 2$ , the answer is 15.)
  - (c) There are  $2 \cdot (2^N - 1) + 1$  subsets that are entirely men or entirely women (I'm counting the emptyset in this collection). This simplifies to  $2^{N+1} - 1$ . (For  $N = 2$ , the answer is 7.)
  - (d) There are  $2^{2N} - [2 \cdot (2^N - 1) + 1]$  subsets that have at least one man and one woman. This simplifies to  $2^{2N} - 2^{N+1} + 1$ . (For  $N = 2$ , the answer is 9.)
  - (e) The proportion is  $\frac{2^{2N} - [2 \cdot (2^N - 1) + 1]}{2^{2N}}$ . This simplifies to  $\frac{2^{2N} - 2^{N+1} + 1}{2^{2N}} = 1 - \frac{1}{2^{N-1}} + \frac{1}{2^{2N}}$ . (For  $N = 2$ , this is  $1 - 1/2 + 1/16 = 9/16$ .)
  - (f) The values of  $N$  for which this proportion is greater than  $1/2$  is 2.
  - (g) The number of subsets that do not include some particular person is  $2^{2N-1}$ . (For  $N = 2$ , this is 8.)
  - (h) The number of subsets that include a particular pair of two people is  $2^{2N-2}$ . (For  $N = 2$ , the answer is 4.)
  - (i) We can line the people up in a row in  $(2N)!$ . (For  $N = 2$ , the answer is  $4!$ , or 24.)
  - (j) We can line the people up in a row with all the men before all the women in  $N! \cdot N!$  ways. (For  $N = 2$ , the answer is 4.)
  - (k) The number of ways we can line them up so that men and women alternate, with a man beginning, is also  $N! \cdot N!$ . (For  $N = 2$ , the answer is 4.)
  - (l) The number of subsets with exactly  $N$  people from  $X$  is " $2N$  choose  $N$ ", or  $\frac{(2N)!}{N! \cdot N!}$ . (For  $N = 2$ , the answer is 6.)
2. Consider the particular set  $X$  with  $X = \{Mary, Sally, Henry, Jim\}$ .
  - (a) List all the subsets... (there are 16)
  - (b) List all the non-empty subsets (there are 15)
  - (c) List all the subsets that are entirely men or entirely women (there are three of each type that are non-empty, and so seven in total - have to include the emptyset).
  - (d) List all the subsets that have at least one man and one woman (there are 9).

- (e) What proportion of the subsets are of mixed gender? There are 16 total subsets, and seven that are single-gender, so nine mixed gender. The proportion is  $9/16$ , which is greater than half.
- (f) How many subsets do not include Jim? There are eight subsets that include Jim, and so eight that do not include Jim.
- (g) How many subsets include both Jim and Henry? There are four such subsets.
- (h) How many ways can we line them up in a row?  $4!$ , or 24.
- (i) How many ways can we line them up in a row with the men coming before the women? Four.
- (j) How many ways can we line them up in a row with the men and women alternating, and a man beginning? Four.
- (k) How many subsets of two people can we create? Four choose two, or 6.