

Spring 2009, Professor Warnow
Logic and Set Theory Problems

1. For each statement, determine if the statement is true for all subsets X of the real numbers, false for all subsets X , or true of some and false of some. (It would be good to provide proofs if you can!)

- (a) $\forall x \in X, \exists y \in X$ s.t. $x < y$
- (b) $\exists y \in X$ s.t. $\forall x \in X - \{y\}, x < y$
- (c) $\exists A \subseteq X$ s.t. $\forall y \in A, \exists z \in X - A, y < z$.
- (d) $\forall A \subseteq X, \forall y \in A, \exists z \in X - A$ s.t. $y < z$
- (e) $\exists A \subseteq X$ s.t. $\exists z \in X - A \forall y \in A, y < z$.
- (f) $\forall A \subseteq X, \exists z \in X - A$ s.t. $\forall y \in A, y < z$

2. For each statement, determine if the statement is true for all graphs $G = (V, E)$, false for all graphs, or true for some and false for some. (It would be good to provide proofs if you can!)

- (a) $\nexists x \in V$ s.t. $\forall y \in V - \{x\}, (x, y) \notin E$.
- (b) $\exists v \in V$ s.t. $\forall y \in V - \{v\}, (y, v) \in E$
- (c) $\forall y \in V \exists v \in V$ s.t. $(y, v) \in E$
- (d) $\forall y \in V \exists v \in V$ s.t. $(y, v) \notin E$
- (e) $\exists v \in V$ s.t. $\forall y \in V - \{v\}, (y, v) \notin E$.
- (f) $\exists A \subseteq V, \exists B \subseteq V,$ s.t. $\forall a \in A, \forall b \in B, (a, b) \in E$
- (g) $\exists A \subseteq V$ s.t. $\forall v \in V - A, \exists a \in A, (a, v) \in E$

3. **Extra Credit:** Let X be a subset of the real numbers which satisfies each of the following properties:

- If $x \in X$ and y is a real number with $x < y$, then $y \in X$.
- If $x \in X$ and y is a real number with $x > y$, then $y \in X$.

Prove that if $X \neq \emptyset$ that X is the entire set of real numbers.