Quiz #1, CS 336, Fall 2009

1. Let $f(n)$ be defined as follows: $f(1) = 3$, and $f(n) = f(n-1)+1$ whenever $n > 1$. We wish to prove that $f(n) = n$ for all $n \geq 1$. **What is wrong with the following "proof" by induction?**

   The inductive hypothesis is that $f(n) = n$ for all $n \geq 1$.
   Now consider $f(n+1)$. By the definition of the function f,
   $f(n+1) = f(n) + 1$.
   By the inductive hypothesis, $f(n) = n$.
   Hence, $f(n+1) = n + 1$, and we are done.

2. Let $g(0) = 1$ and Let $g(n) = g(n-1) + 10n - 5$ if $n > 0$. We wish to prove that $g(n) = 5n^2 + 1$ by induction. **What is wrong with the following proof?**

   The base case is checked and is okay (g(0)=1, and $5n^2 + 1 = 1$ when $n = 0$). The inductive hypothesis is that $g(N) = 5N^2 + 1$ for some N. We now consider $N + 1$.

   - *$g(N + 1) = g(N) + 10N - 5$ by definition.*
   - *By the inductive hypothesis $g(N) = 5N^2 + 1$*
   - *Therefore $g(N + 1) = 5N^2 + 10N - 4$.*
   - *By arithmetic this is equal to $5(N + 1)^2 + 1$.*

3. Let $h(n)$ be defined recursively by:

   - $h(1) = 2$
   - $h(n) = 2h(n-1) - 1$, for $n \geq 2$.

   First give $h(2), h(3), h(4)$, and $h(5)$. Then, use induction to prove that $h(n) \geq 1.5^n$ for all n.