CS345H: Programming Languages

Lecture 13: Type Inference II

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First: Your Project

- Today is the start of your course project
- Goal: Take what we studied and apply it to a project you design yourself
- This is a team project: Teams must be between 3 and 5 students

Possible Topics

- Your goal is to add at least one major feature to the L language
- Some possible examples:
  - Adding type inference to L
  - Speeding up the L interpreter
  - Adding major language features to L
  - Type inference with novel error reporting
  - ...
- Your creativity is the limit

Deliverables & Time line

- Today: Start of project, form teams
- Nov. 13st 12:30pm: Email me a one page proposal for your project as pdf clearly describing what you want to do and list your team members
- Will receive feedback from proposal
- Dec. 11st 12:30pm : Project due. No late days.

Final Deliverables

- Report written in LateX (at least 15 pages) describing clearly what problem you are solving, what choices you made, challenges encountered and your results.
- All your source code in a tar.gz file compiling on Ubuntu
- You will be graded on size of chosen challenge, your solution and your written report
- Since every project is unique, you will get lots of feedback throughout
- If you are passionate about a PL project not related to L, or want to tackle something especially large with more people, etc: Ask!
- Any questions?

Introduction

- Recall for last time: We are inferring types
- Big idea: Replace all concrete type assumptions with type variables
- Collect constraints on these type variables
- Find most general solution for these constraints to deduce types

Quick Refresher

- Lets quickly look again at one example:
  `let f = lambda x.(f x) in f`
- Type derivation:

$$\dfrac{\dfrac{\begin{array}{l}\Gamma[f \leftarrow a_1][x \leftarrow a_2] \vdash f : a_1 \\ \Gamma[f \leftarrow a_1][x \leftarrow a_2] \vdash x : a_2 \\ a_1 = a_2 \rightarrow a_3\end{array}}{\Gamma[f \leftarrow a_1][x \leftarrow a_2] \vdash (f\ x) : a_3}}{\Gamma[f \leftarrow a_1] \vdash \lambda x.(f\ x) : a_1} \quad \Gamma[f \leftarrow a_1] f \vdash : a_1$$
$$\overline{\Gamma \vdash \text{let } f = \lambda x.(f\ x) \text{ in } f : a_1}$$

- Final Type: a_1 under constraint $a_1 = a_2 \rightarrow a_3$
- This yielded constraint system

$$a_1 = a_2 \rightarrow a_3$$
$$a_1$$

Solving Constraints

- Last time, we discussed two substitution rules that allow us to solve such constraints and find the most general solution
- However, the cost of this is quadratic in the number of constraints
- For a large program, this is prohibitive
- Today: How to efficiently solve type constraint systems

Representing Types

- Our type constraint systems are made up of the following three primitives:
  1. Type constants: Int, String
  2. Type variables: α_1, α_2
  3. Function Types: $X \rightarrow Y$
- Observe that $X \rightarrow Y$ is just in-fix notation for $function(X, Y)$
- To solve type constraints more efficiently, we will write $X \rightarrow Y$ also as $function(X, Y)$, but this is just notation

More Efficient Type Inference

- Big Idea: Maintain equivalence classes of types directly
- Equivalence Class: Set of types that must be equal
- Specifically, if we process constraint of the form $X = Y$, we know that X and Y are equal
- In this case, we want to union the equivalence classes of X and Y
- Also, if X and Y are function types of the form $X_1 \rightarrow X_2$ and $Y_1 \rightarrow Y_2$, we also want to union X_1 and Y_1 as well as X_2 and Y_2

Union-Find

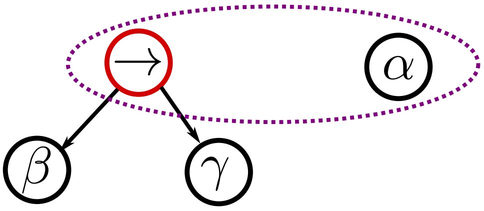
- To maintain equivalence classes directly, we will use the union-find algorithm
- Each set of types is called an equivalence class
- Each set has one element as its representative
- For type inference: If an equivalence contains a type constant or a function type, we will always use this type as the representative.

Union-Find Cont.

- In Union-Find, we have only two operations on equivalence classes:
  1. $Union(s, t)$: This unions the equivalence classes of s and t into one equivalence class
  2. $Find(s)$: This returns the representative of the equivalence class of which s is part of
- Example: Assume following two equivalence classes (representatives in red): $\{int, \alpha\}, \{\beta \rightarrow \gamma, int\}$
- Example: $Union(int, \beta \rightarrow \gamma)$ results in new equivalence class $\{int, \alpha, \beta \rightarrow \gamma\}$
- Example: $Find(\alpha) = int$

Union-Find Representation

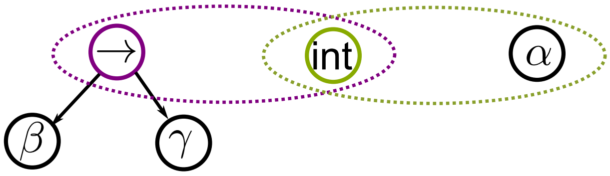
- We will represent equivalence classes as DAGs.
- Example: $\{\beta \to \gamma, \alpha\}$



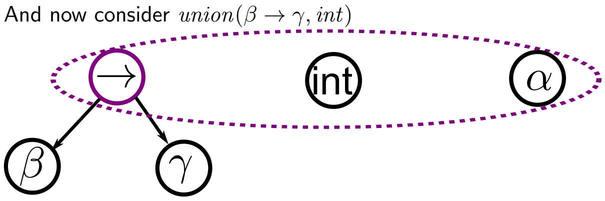
- Conceptually, union will join the dotted areas of two equivalence classes
- And find will return the (red) representative in this class

Union-Find Representation Cont.

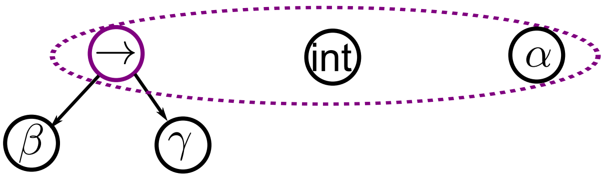
- Consider the following EQs:



- And now consider $union(\beta \to \gamma, int)$



Union-Find Representation



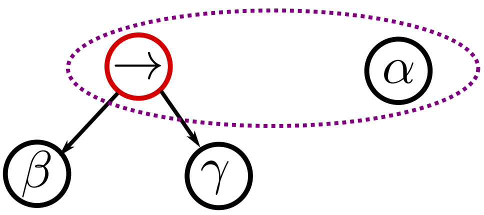
- Question: Is this a possible solution for the type constraints?
- No! If a function type and a constant type ever end up in the same equivalence class, we know that the constraint system has no solution
- We also know constraint system has no solution if Int and $String$ end up in the same EQ

Finding a Solution from the Union-Find DAG

- Assuming we end up with an consistent Union-find DAG, we can read the most general solution right of!
- For each type variable v, simply return $find(v)$
- In other words, the representative of each equivalence class is the most general solution
- Question: Why do we always pick function types or type constants as representatives?
- Question: What happens if a function type and a type constant are in the same equivalence class?

Finding a Solution from the Union-Find DAG

- Example:



- How do we find solution for α?
- $find(\alpha) = \beta \to \gamma$
- What about β?
- Every item is in its own EQ, therefore $find(\beta) = \beta$

Using Union-Find for solving Type Inference Constraints

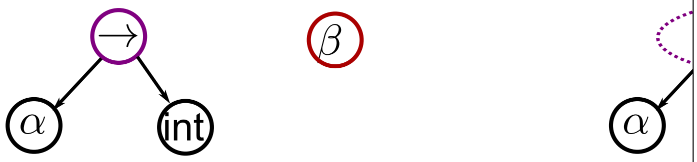
- Initially, all type variables, functions and type constants are in their own equivalence class
- We then apply the following function to each equality in our type constraint:

```
bool unify(m, n) {
  s = find(m); t = find(n);
  if(s == t) return true;
  if(s == s1 → s2 && t == t1 → t2) {
    union(s, t);
    return unify(s1, t1) && unify(s2, t2);
  }
  if(is_variable(s) || is_variable(t)) {
    union(s, t); return true;
  }
  return false; //No solution to type constraints
}
```

Example

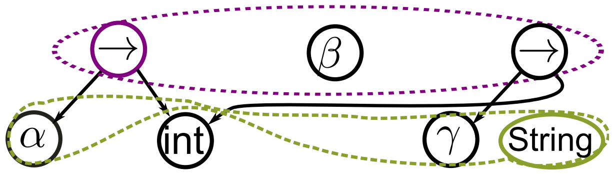
- Consider the following system of type constraints:

$$\begin{aligned} \alpha \rightarrow Int &= \beta \\ \gamma \rightarrow Int &= \beta \\ \gamma &= String \end{aligned}$$



Example Cont

$$\begin{aligned} \alpha \rightarrow Int &= \beta \\ \gamma \rightarrow Int &= \beta \\ \gamma &= String \end{aligned}$$

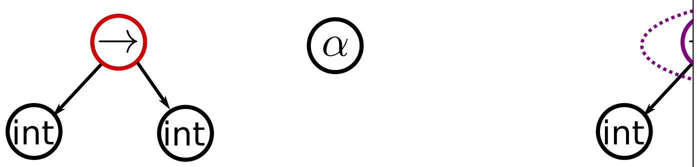


- Solution for α: $find(\alpha) = String$
- Solution for β: $find(\beta) = String \rightarrow Int$
- Solution for γ: $find(\gamma) = String$

Example 2

- Consider the following system of type constraints:

$$\begin{aligned} \alpha &= Int \rightarrow Int \\ \alpha &= String \end{aligned}$$



- Conflict: Unify returns false when trying to unify $Int \rightarrow Int$ and $String$
- Conclusion: This system of type constraints is unsatisfiable

Union-Find

- With this new approach, we can now only process each equality once.
- However, for this to be efficient, union/find must be efficient.
- Key result from algorithms: It is possible to build a data structure for union-find that can find a solution to our sets of type constraints in approximately linear time.
- You can learn about this data structure in Advance Algorithms or Isil's class on automated logical reasoning
- But for our purposes, we will just use this data structure

Type Inference

- If we use Union-Find, we can make type inference practical on real programs
- This style of polymorphic type inference we studied is known as Hindley-Milner type inference
- Type inference is at the core of languages such as OCAML and Haskell
- Type inference is increasingly moving to main-stream languages
  - New C++11 standard
  - Java 7

Type Inference and Errors

- We saw that we can detect all errors easily when doing type inference
- Specifically, every error resulted from unifying two equivalence classes that could not be unified.
- Example: Trying to unify $String$ and $\alpha \rightarrow Int$
- But how do we report this error to programmers?

Error Reporting

- Consider again the example: $String$ and $\alpha \to Int$.
- Option 1: Output message: $String$ and $\alpha \to Int$ cannot be unified.
- Is this helpful?
- Obvious problems:
  - Not associated with any source location
  - Understanding typing errors requires understanding type inference

Error Reporting Cont.

- Improvement used in practice: Associate expression/source location with type constraint.
- Message can now at least contain the program expressions that evaluate to $String$ and $\alpha \to Int$
- But the actual error in your program may be arbitrarily far from these locations!
- Typical OCaml error:
  "`At line 37: Expected expression of type 'a -> 'a but found expression of type 'a -> 'b`"
- To fix this, you need to understand all the reasoning steps that happened during type inference
- Most likely, the problem did not originate at line 37!

Type Annotations

- Most common technique for mitigating these difficulties: Allow type annotations
- Type annotations allow you explicitly declare types even though the compiler can infer them automatically
- Idea: If you encounter a type error you do not understand, you give the type you expect to the expressions involved in this error and re-run the type checker
- You will now get a new type error in a different location
- You repeat this process until you fixed your type error

Type Annotations Drawbacks

- However, this approach still has substantial drawbacks:
  - You often need many annotations to find the source of type errors
  - You can only annotate successfully if you understand polymorphic type inference
  - You often end up with a program that is almost completely type annotated!

Type Inference in the Real World

- Despite these difficulties, there are many real languages that support full type inference.
- Examples: OCaml, Haskell, F#
- Slogan on Type Inference: The ease of dynamic typing with the speed an guarantees of a static type system
- This claim is true, but real problems with explaining typing errors to programmers
- Explaining typing errors better is also an active research area!

Type Inference in the Real World Cont.

- Alternative approach taken by more main-stream languages recently: local type inference
- In local type inference, types are only inferred within one function, but must be fully annotated at function boundaries.
- Goal: Make it easier for programmers to diagnose type errors (and make type inference tractable in the imperative setting)

Example of local type inference

- C++ supports some forms of local type inference.
- First Example: templates
- A STL pair is templatized over the type of the first and second element
- You declare a pair as: `pair<int, string> p(3, "duck");`
- However, if you call a function that takes a pair, the compiler will infer the template type for you in some cases:
- Example: `edit_pair(p)` instead of `edit_pair<pair<int, string> >(p)`

Example of local type inference

- The new C++11 standard supports much more expressive local type inference
- This is done using the `auto` keyword
- Example using iterator: `vector<int> v;`

```
...
for(vector<int>::iterator it = v.begin(); it !=
v.end(); it++) ...
```

- Example using iterator with new auto keyword:

```
vector<int> v;
...
for(auto it = v.begin(); it != v.end(); it++) ...
```


Type Inference in C++

- The `auto` keyword really just says "do type inference on this expression and figure the type out"
- Very convenient, local feature that is also creeping into languages such as C# and Java
- You will see more of this in the future

Conclusion

- We saw how to use Union-Find to make type inference scalable
- This formulation is one of the classic and elegant results in programming languages, known as Hindley-Milner type inference
- Type inference is most likely coming to your favorite language in the near future, if it is not already there!