First: Your Project

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- **Goal:** Take what we studied and apply it to a project you design yourself
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- **Goal:** Take what we studied and apply it to a project you design yourself
- This is a **team project:** Teams must be between 3 and 5 students
Possible Topics

- Your goal is to add at least one major feature to the L language
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- Some possible examples:
  - Adding type inference to L
  - Speeding up the L interpreter
  - Adding major language features to L
  - Type inference with novel error reporting
Possible Topics

▶ Your goal is to add at least one major feature to the L language

▶ Some possible examples:
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▶ Some possible examples:
  ▶ Adding type inference to L
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  ▶ Type inference with novel error reporting
  ▶ …

▶ Your creativity is the limit
Deliverables & Time line

- **Today:** Start of project, form teams
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- **Nov. 13st 12:30pm**: Email me a one page proposal for your project as pdf clearly describing what you want to do and list your team members
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- Will receive feedback from proposal

- **Dec. 11st 12:30pm**: Project due. No late days.
Final Deliverables

- Report written in LaTeX (at least 15 pages) describing clearly what problem you are solving, what choices you made, challenges encountered and your results.
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- If you are passionate about a PL project not related to L, or want to tackle something especially large with more people, etc: Ask!
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- If you are passionate about a PL project not related to L, or want to tackle something especially large with more people, etc: Ask!

- Any questions?
Introduction

- Recall for last time: We are inferring types
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- Big idea: Replace all concrete type assumptions with type variables
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Big idea: Replace all concrete type assumptions with type variables

Collect constraints on these type variables
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Big idea: Replace all concrete type assumptions with type variables

Collect constraints on these type variables

Find most general solution for these constraints to deduce types
Quick Refresher

- Lets quickly look again at one example:
  let f = lambda x.(f x) in f

▶ Type derivation:
\[ \Gamma \vdash f : a_1 \]
\[ \Gamma \vdash x : a_2 \]
\[ a_1 = a_2 \rightarrow a_3 \]
\[ \Gamma \vdash (f x) : a_3 \]
\[ \Gamma \vdash \lambda x.(f x) : a_1 \]
\[ \Gamma \vdash f : a_1 \]

▶ Final Type:
\[ a_1 \] under constraint \[ a_1 = a_2 \rightarrow a_3 \]

▶ This yielded constraint system
\[ a_1 = a_2 \rightarrow a_3 \]
Quick Refresher

- Lets quickly look again at one example:
  ```
  let f = lambda x.(f x) in f
  ```

- Type derivation:

  $\Gamma[f \leftarrow a_1][x \leftarrow a_2] \vdash f : a_1$
  
  $\Gamma[f \leftarrow a_1][x \leftarrow a_2] \vdash x : a_2$
  
  $a_1 = a_2 \rightarrow a_3$

  $\Gamma[f \leftarrow a_1][x \leftarrow a_2] \vdash (f \ x) : a_3$

  $\Gamma[f \leftarrow a_1][x \leftarrow a_2] \vdash \lambda x.(f \ x) : a_1$

  $\Gamma[f \leftarrow a_1]f \vdash: a_1$

  $\Gamma \vdash \text{let } f = \lambda x.(f \ x) \text{ in } f : a_1$
Quick Refresher

- Lets quickly look again at one example:
  
  ```
  let f = lambda x.(f x) in f
  ```

- Type derivation:

  
  \[
  \begin{align*}
  \Gamma[f \leftarrow a_1][x \leftarrow a_2] & \vdash f : a_1 \\
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  a_1 = a_2 \rightarrow a_3 & \\
  \Gamma[f \leftarrow a_1][x \leftarrow a_2] & \vdash (f \ x) : a_3 \\
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  \Gamma & \vdash \text{let } f = \lambda x.(f \ x) \text{ in } f : a_1
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  \]

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let f = lambda x. (f x) in f

▶ Type derivation:

\[
\begin{align*}
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\Gamma[f \leftarrow a_1][x \leftarrow a_2] & \vdash x : a_2 \\
\Gamma[f \leftarrow a_1][x \leftarrow a_2] & \vdash (f \, x) : a_3 \\
\Gamma[f \leftarrow a_1] & \vdash \lambda x. (f \, x) : a_1 \\
\Gamma & \vdash \text{let } f = \lambda x. (f \, x) \text{ in } f : a_1 \\
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▶ Final Type: \( a_1 \) under constraint \( a_1 = a_2 \rightarrow a_3 \)

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\[
\begin{align*}
a_1 &= a_2 \rightarrow a_3 \\
a_1
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Solving Constraints

- Last time, we discussed two substitution rules that allow us to solve such constraints and find the most general solution.
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- Today: How to efficiently solve type constraint systems.
Representing Types

- Our type constraint systems are made up of the following three primitives:
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  1. Type constants:
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  1. Type constants: Int, String
  2. Type variables:

  Observe that \( X \rightarrow Y \) is just in-fix notation for function \((X, Y)\).
  To solve type constraints more efficiently, we will write \( X \rightarrow Y \) also as \( \text{function}\ (X, Y)\), but this is just notation.
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More Efficient Type Inference

- **Big Idea:** Maintain equivalence classes of types directly
More Efficient Type Inference

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▶ **Equivalence Class:** Set of types that must be equal
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- Specifically, if we process constraint of the form $X = Y$, we know that $X$ and $Y$ are equal
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More Efficient Type Inference

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- **Equivalence Class:** Set of types that must be equal

- Specifically, if we process constraint of the form $X = Y$, we know that $X$ and $Y$ are equal

- In this case, we want to **union** the equivalence classes of $X$ and $Y$

- Also, if $X$ and $Y$ are function types of the form $X_1 \rightarrow X_2$ and $Y_1 \rightarrow Y_2$, we also want to union $X_1$ and $Y_1$ as well as $X_2$ and $Y_2$
Union-Find

- To maintain equivalence classes directly, we will use the union-find algorithm
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- For type inference: If an equivalence contains a type constant or a function type, we will always use this type as the representative.
Union-Find Cont.

- In Union-Find, we have only two operations on equivalence classes:

  1. **Union** \((s, t)\): This unites the equivalence classes of \(s\) and \(t\) into one equivalence class.
  2. **Find** \((s)\): This returns the representative of the equivalence class of which \(s\) is part of.

Example: Assume the following two equivalence classes (representatives in red):

\[
\{\text{int}, \alpha\}, \{\beta \rightarrow \gamma, \text{int}\}
\]

Example: **Union** \((\text{int}, \beta \rightarrow \gamma)\) results in a new equivalence class \(\{\text{int}, \alpha, \beta \rightarrow \gamma\}\).

Example: **Find** \((\alpha)\) = \text{int}.
Union-Find Cont.

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Example: Assume following two equivalence classes (representatives in red): \( \{ \text{int}, \alpha \}, \{ \beta \rightarrow \gamma, \text{int} \} \)

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Example: \( \text{Find}(\alpha) = \text{int} \)
Union-Find Representation

- We will represent equivalence classes as DAGs.
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- Conceptually, union will join the dotted areas of two equivalence classes
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- Example: \( \{ \beta \rightarrow \gamma, \alpha \} \)

- Conceptually, union will join the dotted areas of two equivalence classes

- And find will return the (red) representative in this class
Union-Find Representation Cont.

▶ Consider the following EQs:
Union-Find Representation Cont.

- Consider the following EQs:

\[
\begin{array}{c}
\beta \\
\rightarrow \\
\gamma \\
\end{array} 
\]

And now consider \( \text{union}(\beta \rightarrow \gamma, \text{int}) \).
Union-Find Representation Cont.

- Consider the following EQs:

  And now consider $\text{union}(\beta \rightarrow \gamma, \text{int})$
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  - And now consider \( \text{union}(\beta \rightarrow \gamma, \text{int}) \)
Question: Is this a possible solution for the type constraints?
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No! If a function type and a constant type ever end up in the same equivalence class, we know that the constraint system has no solution.
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No! If a function type and a constant type ever end up in the same equivalence class, we know that the constraint system has no solution.

We also know constraint system has no solution if Int and String end up in the same EQ.
Finding a Solution from the Union-Find DAG

- Assuming we end up with a consistent Union-find DAG, we can read the most general solution right of!

  For each type variable $v$, simply return $\text{find}(v)$.

  In other words, the representative of each equivalence class is the most general solution.

Question: Why do we always pick function types or type constants as representatives?

Question: What happens if a function type and a type constant are in the same equivalence class?
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- **Question**: Why do we always pick function types or type constants as representatives?

- **Question**: What happens if a function type and a type constant are in the same equivalence class?
Finding a Solution from the Union-Find DAG

Example:

\[ \text{How do we find solution for } \alpha? \]

\[ \text{find}(\alpha) = \beta \rightarrow \gamma \]

What about \( \beta \)?

Every item is in its own EQ, therefore

\[ \text{find}(\beta) = \beta \]
Finding a Solution from the Union-Find DAG

Example:

How do we find solution for $\alpha$?

Every item is in its own EQ, therefore $\text{find}(\beta) = \beta$. 

What about $\beta$?
Finding a Solution from the Union-Find DAG

Example:

How do we find solution for $\alpha$?

$\text{find}(\alpha) =$
Finding a Solution from the Union-Find DAG

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Finding a Solution from the Union-Find DAG

- Example:

  - How do we find solution for $\alpha$?
  - $\text{find}(\alpha) = \beta \rightarrow \gamma$
  - What about $\beta$?
Finding a Solution from the Union-Find DAG

Example:

- How do we find solution for $\alpha$?
  - $\text{find}(\alpha) = \beta \rightarrow \gamma$
- What about $\beta$?
- Every item is in its own EQ, therefore $\text{find}(\beta) = \beta$
Using Union-Find for solving Type Inference Constraints

- Initially, all type variables, functions and type constants are in their own equivalence class

```c
bool unify(m, n) {
    s = find(m); t = find(n);
    if(s == t) return true;
    if(s == s1 \rightarrow s2 && t == t1 \rightarrow t2) {
        union(s, t);
        return unify(s1, t1) && unify(s2, t2);
    }
    if(is_variable(s) || is_variable(t)) {
        union(s, t); return true;
    }
    return false; //No solution to type constraints
}
```
Using Union-Find for solving Type Inference Constraints

- Initially, all type variables, functions and type constants are in their own equivalence class

- We then apply the following function to each equality in our type constraint:

  ```
  bool unify(m, n) {
    s = find(m); t = find(n);
    if(s == t) return true;
    if(s == s1 → s2 && t == t1 → t2) {
      union(s, t);
      return unify(s1, t1) && unify(s2, t2);
    }
    if(is_variable(s) || is_variable(t)) {
      union(s, t); return true;
    }
    return false; //No solution to type constraints
  }
  ```
Example

- Consider the following system of type constraints:

\[
\alpha \rightarrow Int = \beta \\
\gamma \rightarrow Int = \beta \\
\gamma = String
\]
Example

Consider the following system of type constraints:

\[
\begin{align*}
\alpha &\rightarrow \text{Int} = \beta \\
\gamma &\rightarrow \text{Int} = \beta \\
\gamma &= \text{String}
\end{align*}
\]
Consider the following system of type constraints:

\[
\begin{align*}
\alpha \rightarrow \text{Int} &= \beta \\
\gamma \rightarrow \text{Int} &= \beta \\
\gamma &= \text{String}
\end{align*}
\]
Example

Consider the following system of type constraints:

\[ \alpha \rightarrow Int = \beta \]
\[ \gamma \rightarrow Int = \beta \]
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Consider the following system of type constraints:

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\alpha \rightarrow \text{Int} = \beta \\
\gamma \rightarrow \text{Int} = \beta \\
\gamma = \text{String}
\]
Consider the following system of type constraints:

\[
\begin{align*}
\alpha & \rightarrow \text{Int} \quad = \quad \beta \\
\gamma & \rightarrow \text{Int} \quad = \quad \beta \\
\gamma & \quad = \quad \text{String}
\end{align*}
\]
Example

Consider the following system of type constraints:

\[
\begin{align*}
\alpha & \rightarrow \text{Int} = \beta \\
\gamma & \rightarrow \text{Int} = \beta \\
\gamma & = \text{String}
\end{align*}
\]
Consider the following system of type constraints:

\[ \alpha \rightarrow \text{Int} = \beta \]
\[ \gamma \rightarrow \text{Int} = \beta \]
\[ \gamma = \text{String} \]
Example Cont

\[
\alpha \rightarrow Int = \beta \\
\gamma \rightarrow Int = \beta \\
\gamma = String
\]

- Solution for \(\alpha\):

- Solution for \(\beta\):

- Solution for \(\gamma\):

...
Example Cont

\[ \alpha \rightarrow \text{Int} = \beta \]
\[ \gamma \rightarrow \text{Int} = \beta \]
\[ \gamma = \text{String} \]

▶ Solution for \( \alpha \): \( \text{find}(\alpha) = \)
Example Cont

\[ \alpha \rightarrow \text{Int} = \beta \]
\[ \gamma \rightarrow \text{Int} = \beta \]
\[ \gamma = \text{String} \]

- Solution for \( \alpha \): \( \text{find}(\alpha) = \text{String} \)
Example Cont

\[ \alpha \rightarrow \text{Int} = \beta \]
\[ \gamma \rightarrow \text{Int} = \beta \]
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Solution for \( \alpha \): \( \text{find}(\alpha) = \text{String} \)

Solution for \( \beta \):
Example Cont

\[ \alpha \rightarrow \text{Int} = \beta \]
\[ \gamma \rightarrow \text{Int} = \beta \]
\[ \gamma = \text{String} \]

- Solution for \( \alpha \): \( \text{find}(\alpha) = \text{String} \)

- Solution for \( \beta \): \( \text{find}(\beta) = \)
Example Cont

\[ \alpha \rightarrow \text{Int} = \beta \]
\[ \gamma \rightarrow \text{Int} = \beta \]
\[ \gamma = \text{String} \]

- Solution for $\alpha$: $\text{find}(\alpha) = \text{String}$
- Solution for $\beta$: $\text{find}(\beta) = \text{String} \rightarrow \text{Int}$
Example Cont

\[ \alpha \rightarrow \text{Int} = \beta \]
\[ \gamma \rightarrow \text{Int} = \beta \]
\[ \gamma = \text{String} \]

Solution for \( \alpha \): \( \text{find}(\alpha) = \text{String} \)

Solution for \( \beta \): \( \text{find}(\beta) = \text{String} \rightarrow \text{Int} \)

Solution for \( \gamma \):
Example Cont

\[ \alpha \rightarrow Int = \beta \]
\[ \gamma \rightarrow Int = \beta \]
\[ \gamma = String \]

- Solution for \( \alpha \): \( find(\alpha) = String \)
- Solution for \( \beta \): \( find(\beta) = String \rightarrow Int \)
- Solution for \( \gamma \): \( find(\gamma) = \)
Example Cont

\[ \alpha \to \text{Int} = \beta \]
\[ \gamma \to \text{Int} = \beta \]
\[ \gamma = \text{String} \]

▶ Solution for \(\alpha\): \(\text{find}(\alpha) = \text{String}\)

▶ Solution for \(\beta\): \(\text{find}(\beta) = \text{String} \to \text{Int}\)

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Consider the following system of type constraints:

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- But for our purposes, we will just use this data structure.
Type Inference

- If we use Union-Find, we can make type inference practical on real programs
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Specifically, every error resulted from unifying two equivalence classes that could not be unified. Example: Trying to unify `String` and `α → Int`.

But how do we report this error to programmers?
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Error Reporting Cont.

- Improvement used in practice: Associate expression/source location with type constraint.

But the actual error in your program may be arbitrarily far from these locations!

Typical OCaml error:

"At line 37: Expected expression of type 'a -> 'a but found expression of type 'a -> 'b"

To fix this, you need to understand all the reasoning steps that happened during type inference.

Most likely, the problem did not originate at line 37!
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Type Annotations Drawbacks

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- You often need many annotations to find the source of type errors
- You can only annotate successfully if you understand polymorphic type inference
- You often end up with a program that is almost completely type annotated!
Type Inference in the Real World

- Despite these difficulties, there are many real languages that support full type inference.
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- Explaining typing errors better is also an active research area!
Type Inference in the Real World Cont.

- Alternative approach taken by more main-stream languages recently: **local type inference**
Type Inference in the Real World Cont.

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- In local type inference, types are only inferred within one function, but must be fully annotated at function boundaries.
Type Inference in the Real World Cont.

- Alternative approach taken by more main-stream languages recently: local type inference

- In local type inference, types are only inferred within one function, but must be fully annotated at function boundaries.

- **Goal**: Make it easier for programmers to diagnose type errors (and make type inference tractable in the imperative setting)
Example of local type inference

- C++ supports some forms of local type inference.
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- Example: `edit_pair(p)` instead of `edit_pair<pair<int, string>> >(p)`
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Example using iterator:  
```cpp
vector<int> v;
...
for(vector<int>::iterator it = v.begin(); it != v.end(); it++) ...
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- **Example using iterator:**
  ```
  vector<int> v;
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  for(vector<int>::iterator it = v.begin(); it != v.end(); it++) ...
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- **Example using iterator with new auto keyword:**
  ```
  vector<int> v;
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  ```
Type Inference in C++

- The `auto` keyword really just says “do type inference on this expression and figure the type out”
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- Very convenient, local feature that is also creeping into languages such as C# and Java
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Very convenient, local feature that is also creeping into languages such as C# and Java

You will see more of this in the future
Conclusion

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- This formulation is one of the classic and elegant results in programming languages, known as Hindley-Milner type inference.

- Type inference is most likely coming to your favorite language in the near future, if it is not already there!