CS345H: Programming Languages

Lecture 13: Type Inference II

Thomas Dillig

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- This is a team project: Teams must be between 3 and 5 students

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- Your creativity is the limit

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- Any questions?

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- Big idea: Replace all concrete type assumptions with type variables
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- This yielded constraint system

$$\begin{array}{l} a_1 = a_2 \rightarrow a_3 \\ a_1 \end{array}$$

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- Today: How to efficiently solve type constraint systems

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- To solve type constraints more efficiently, we will write $X \rightarrow Y$ also as $function(X, Y)$, but this is just notation

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- Specifically, if we process constraint of the form $X = Y$, we know that X and Y are equal
- In this case, we want to union the equivalence classes of X and Y
- Also, if X and Y are function types of the form $X_1 \to X_2$ and $Y_1 \to Y_2$, we also want to union X_1 and Y_1 as well as X_2 and Y_2

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- For type inference: If an equivalence contains a type constant or a function type, we will always use this type as the representative.

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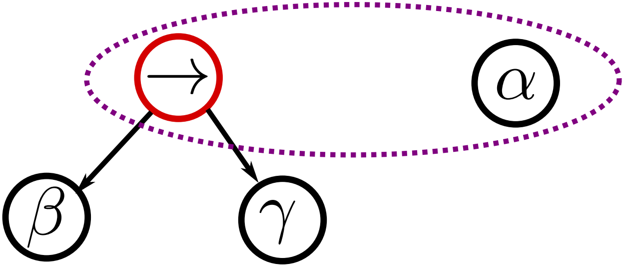

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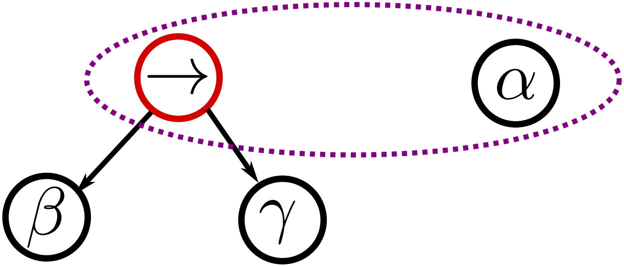

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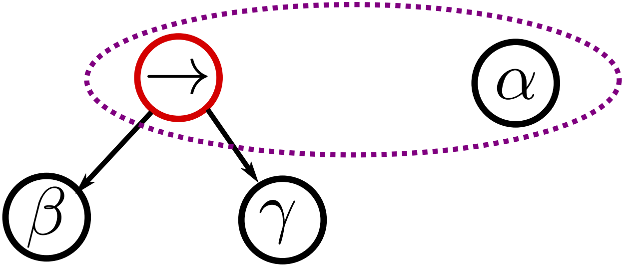

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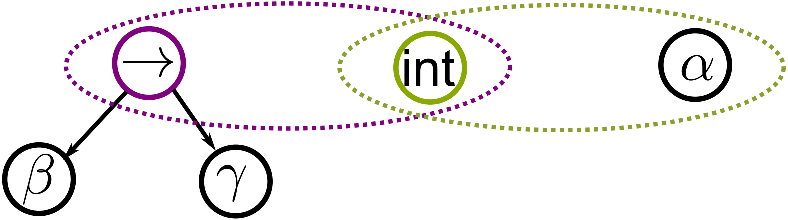

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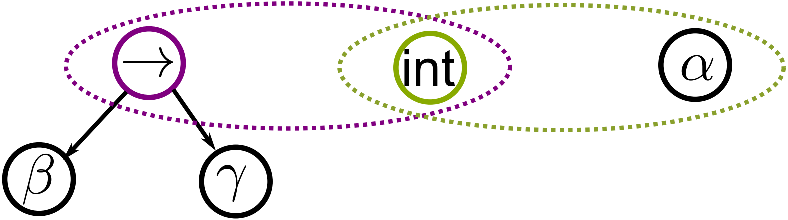

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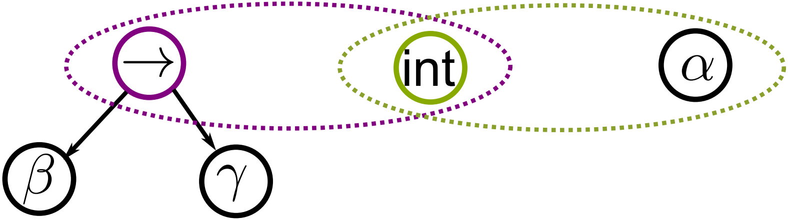

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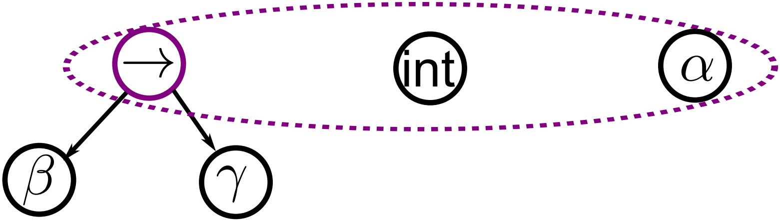

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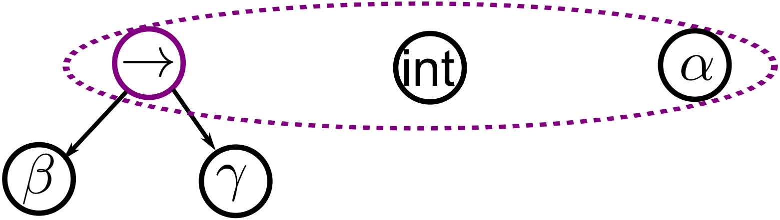

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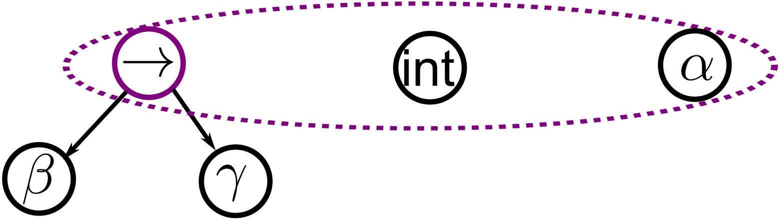

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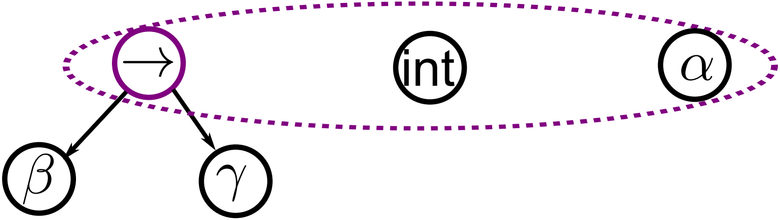

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- No! If a function type and a constant type ever end up in the same equivalence class, we know that the constraint system has no solution
- We also know constraint system has no solution if Int and $String$ end up in the same EQ

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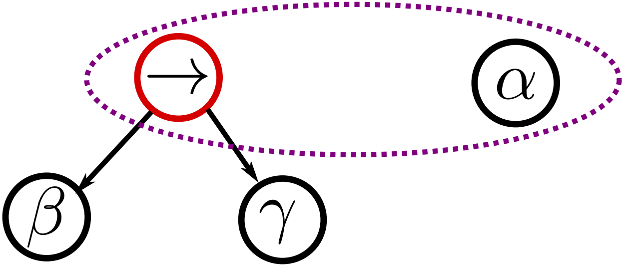

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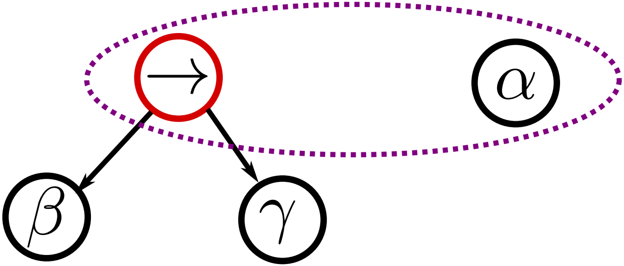

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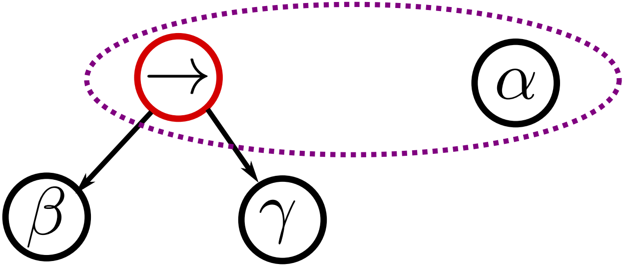

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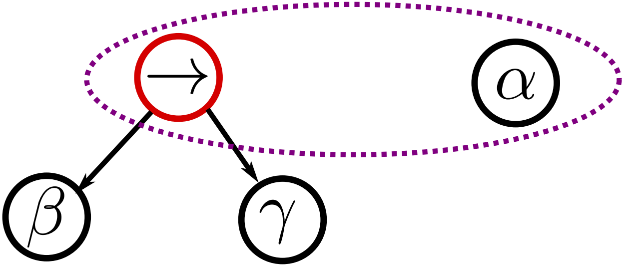

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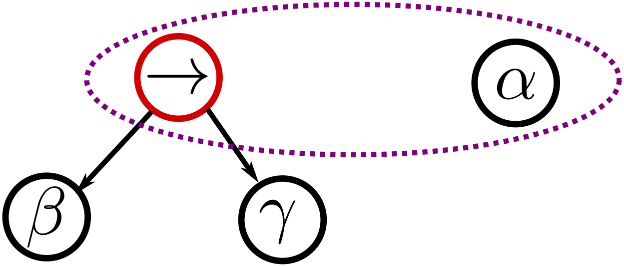

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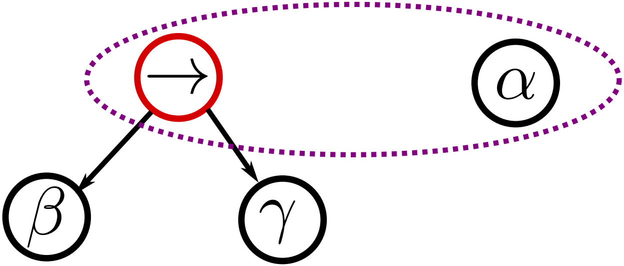

- How do we find solution for α?
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- Every item is in its own EQ, therefore $\mathit{find}(\beta) = \beta$

Using Union-Find for solving Type Inference Constraints

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- We then apply the following function to each equality in our type constraint:

```
bool unify(m, n) {
  s = find(m); t = find(n);
  if(s == t) return true;
  if(s == s1 → s2 && t == t1 → t2) {
    union(s, t);
    return unify(s1, t1) && unify(s2, t2);
  }
  if(is_variable(s) || is_variable(t)) {
    union(s, t); return true;
  }
  return false; //No solution to type constraints
}
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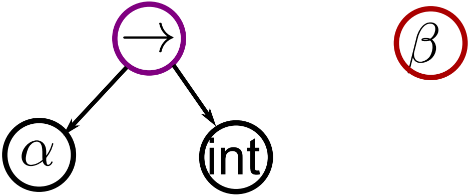

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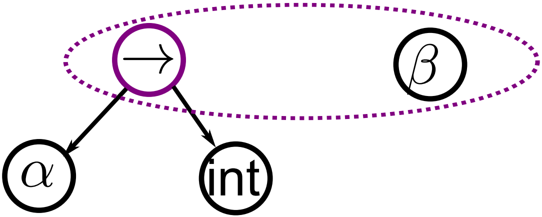

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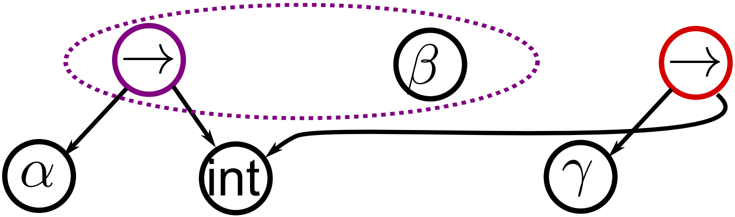

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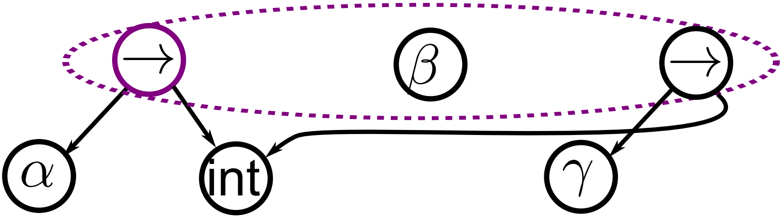

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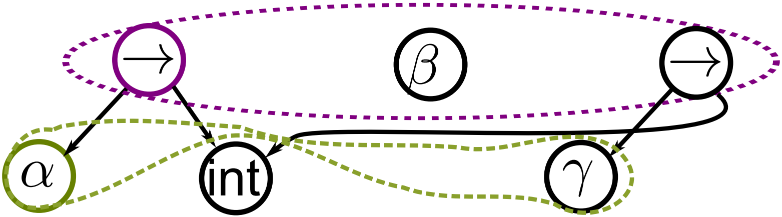

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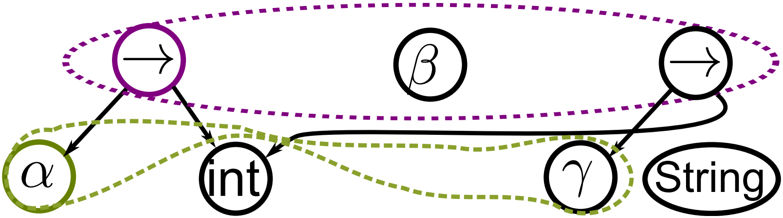

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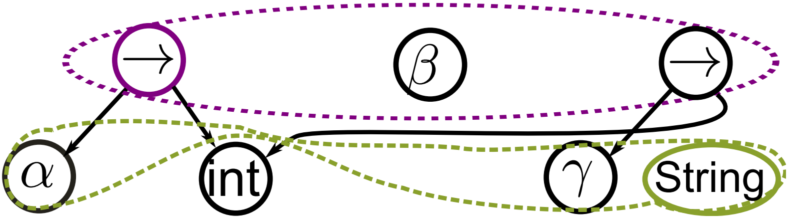

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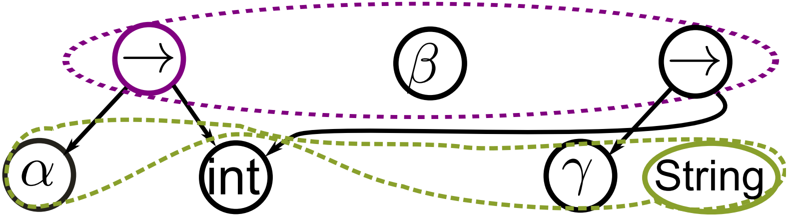

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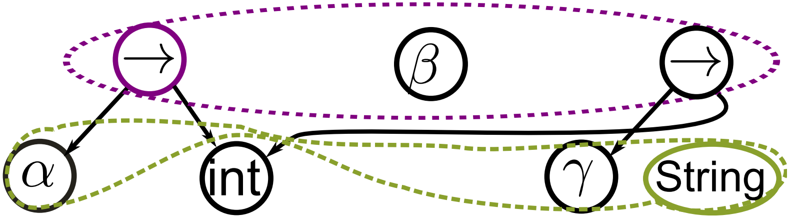

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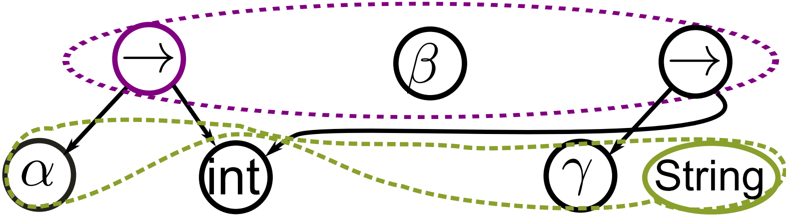

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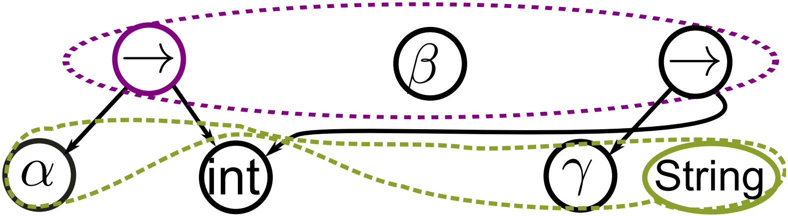

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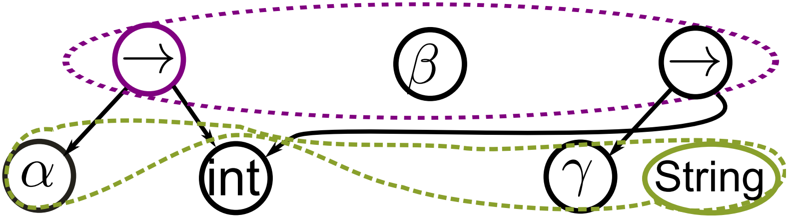

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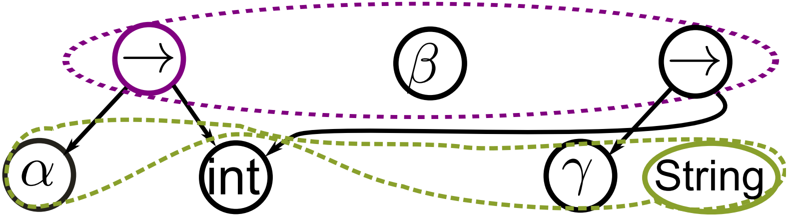

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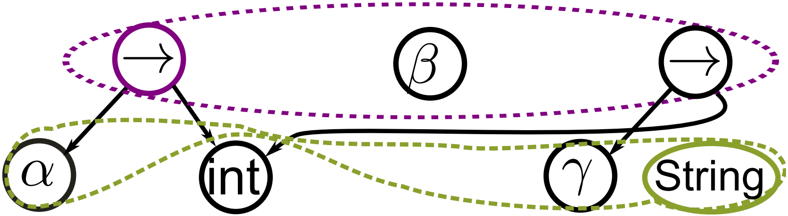

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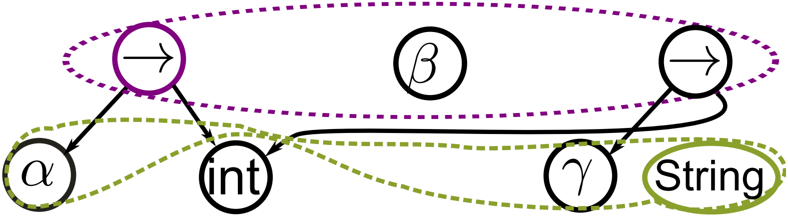

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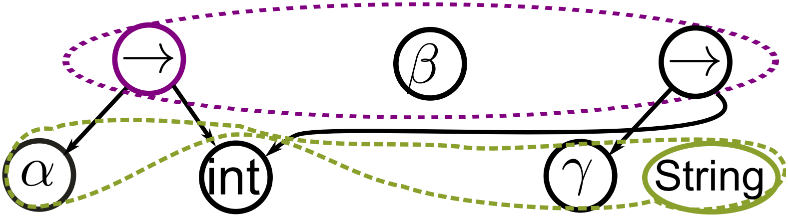

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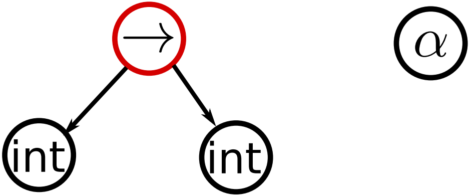

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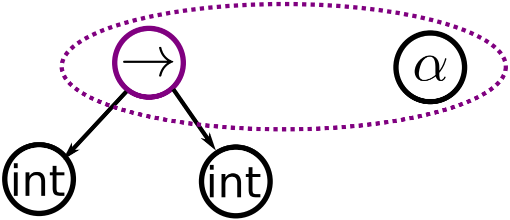

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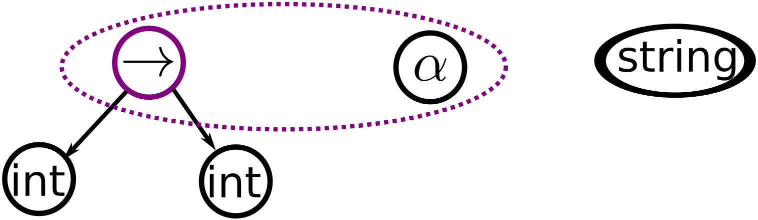

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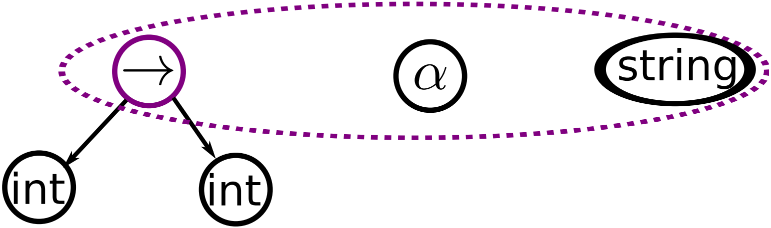

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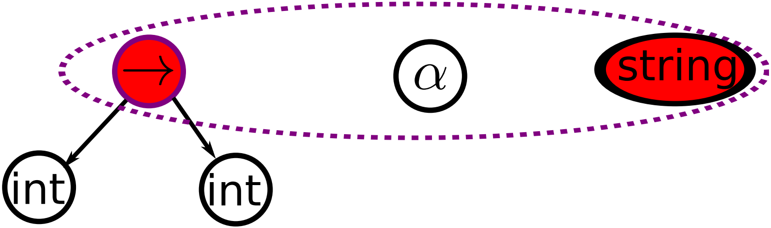

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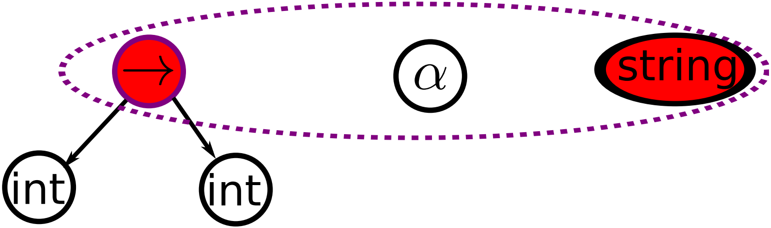

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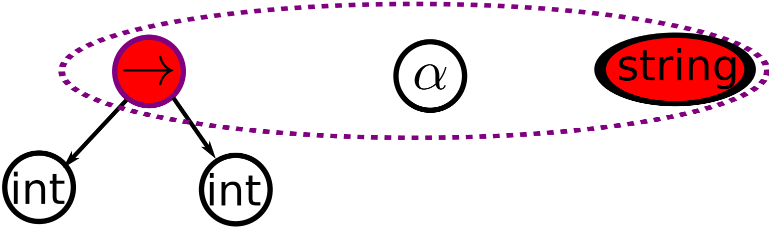

- Conflict: Unify returns false when trying to unify $Int \to Int$ and $String$
- Conclusion: This system of type constraints is unsatisfiable

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- But how do we report this error to programmers?

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- Explaining typing errors better is also an active research area!

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- Alternative approach taken by more main-stream languages recently: local type inference
- In local type inference, types are only inferred within one function, but must be fully annotated at function boundaries.
- Goal: Make it easier for programmers to diagnose type errors (and make type inference tractable in the imperative setting)

Example of local type inference

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- Example: `edit_pair(p)` instead of `edit_pair<pair<int, string> >(p)`

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- You will see more of this in the future

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- We saw how to use Union-Find to make type inference scalable
- This formulation is one of the classic and elegant results in programming languages, known as Hindley-Milner type inference
- Type inference is most likely coming to your favorite language in the near future, if it is not already there!