

CS345H: Programming Languages

Lecture 4: Implementation of Lexical Analysis

Thomas Dillig

Announcements

- ▶ WA1 and PA0 are due **Today**

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- ▶ WA2 and PA1 out today :-)

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- ▶ WA2 and PA1 out today :-)
- ▶ If you are not very, very busy right now, **get started now**

Outline

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- ▶ Today: How to recognize strings matching regular expressions using finite automata.
- ▶ We will see determinist finite automata (DFAs) and non-deterministic finite automata (NFAs)
- ▶ **High-level story:** RegEx $\rightarrow$ NFA $\rightarrow$ DFA $\rightarrow$ Tables

Regular Expressions in Lexical Specifications

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- ▶ But yes/no answer is not enough!
- ▶ We really want to partition input into tokens
- ▶ We adapt regular expressions for this goal

Regular Expressions to Lexical Specifications (1)

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- ▶ $R = \text{Integer constant} + \text{Identifier} + \text{Lambda} + \dots$

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- ▶ **Step 3:** For each $1 \leq i \leq n$ check $x_1 \dots x_j \in L(R)$ for some j
- ▶ Then, remove $x_1 \dots x_j$ from input and repeat

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- ▶ **Example:** identifier = letter (letter + digit)*, if = 'i' 'f'
- ▶ **Rule:** Pick longest possible string in $L(R)$
- ▶ This is known as “maximal munch”

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- ▶ Rule: Use rule listed first
- ▶ This is how "if" is matched as a keyword, not identifier

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- ▶ **Better Solution:** Write a rule matching all “bad” strings
- ▶ **Question:** What kind of rule and where to place it?

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- ▶ Next: How to decide if $s \in L(R)$

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 - ▶ A set of transitions state $\rightarrow^{\text{input}}$ state

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Finite Automata as State Graphs

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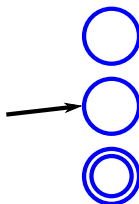
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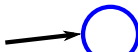
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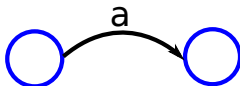
The start state:



An accepting state:



A transition:

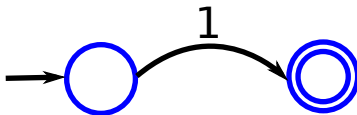


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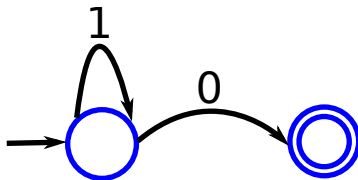
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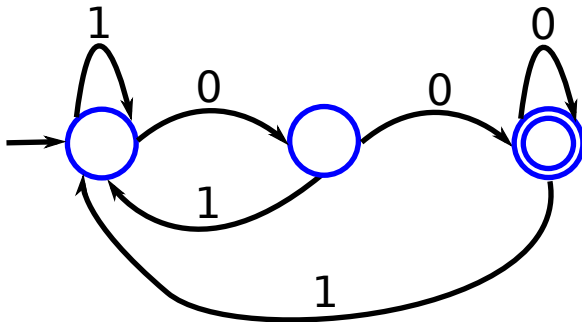
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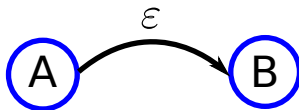


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- ▶ Another kind of transition: ϵ -moves

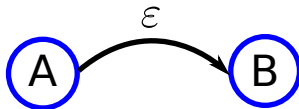
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- ▶ Machine can move from state A to B without reading any input

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- ▶ NFAs can choose:
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 - ▶ Which one of multiple transitions for a single input to take

Acceptance of NFAs

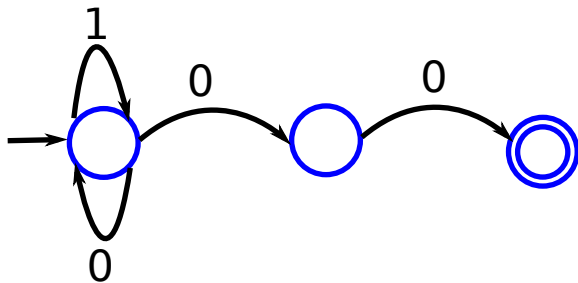
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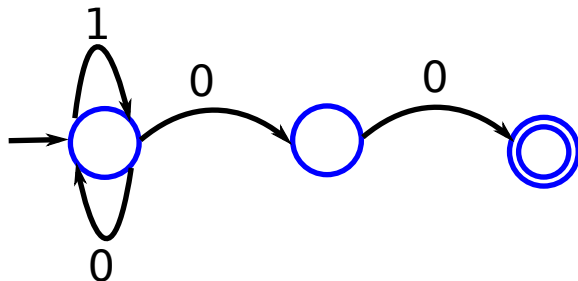
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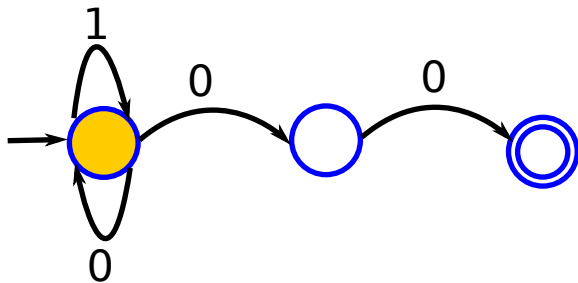
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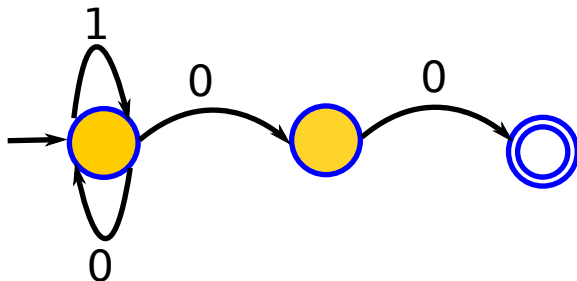
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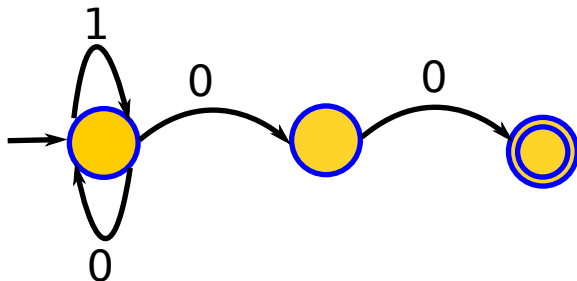
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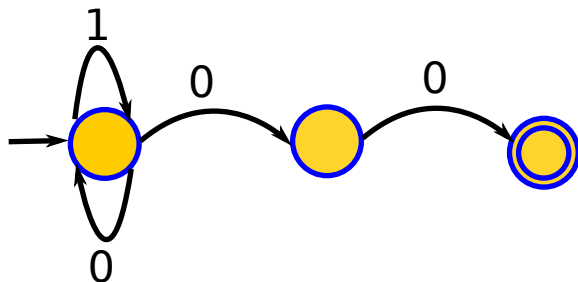
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- ▶ **Rule:** NFA accepts if it can get to a final state

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- ▶ **Result:** DFAs can be exponentially larger than NFA recognizing same language

Regular Expressions to Finite Automata

- ▶ High-Level Sketch:

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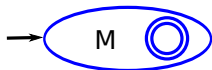
⇒ Lexer

Regular Expressions to NFA (1)

- ▶ For each kind of regular expression, define an NFA and combine

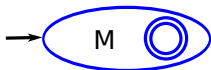
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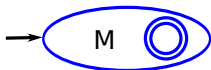
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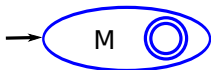


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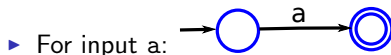
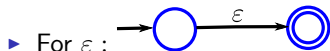


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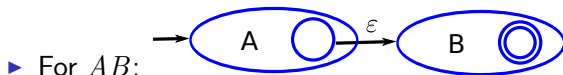
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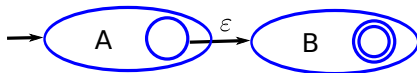


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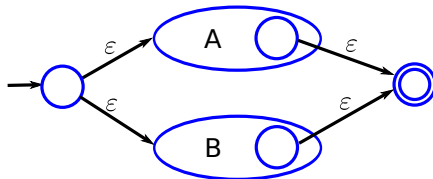


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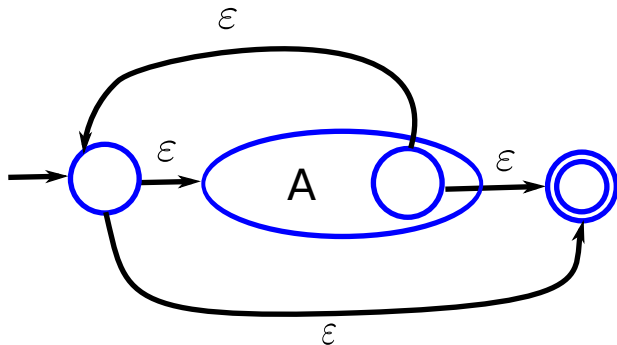
► For AB :



► For $A + B$:



Regular Expressions to NFA (3)



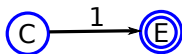
► For A^* :

Example of Regular Expression to NFA conversion

- ▶ Consider the regular expression $(1 + 0)^*1$

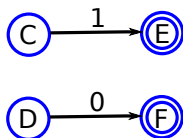
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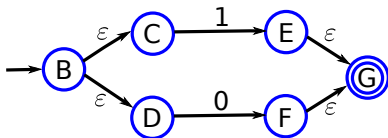
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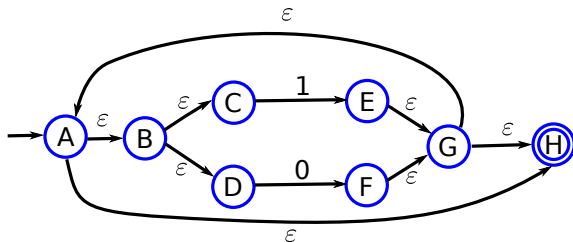
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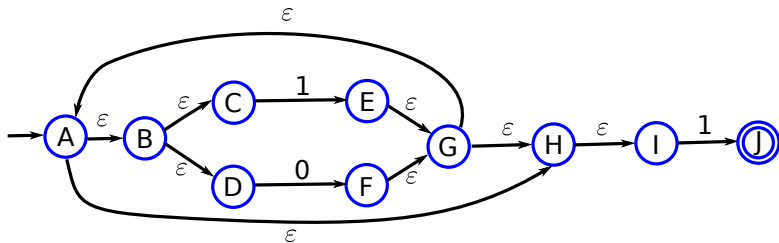
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- ▶ States in the DFA $\Rightarrow$ all (reachable) subsets of states in the NFA

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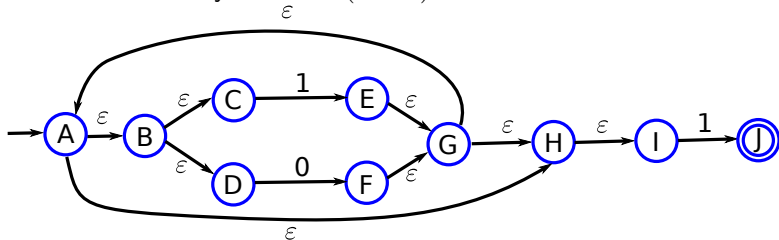
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NFA to DFA: Example

Recall our friendly NFA for $(1 + 0)^*1$:

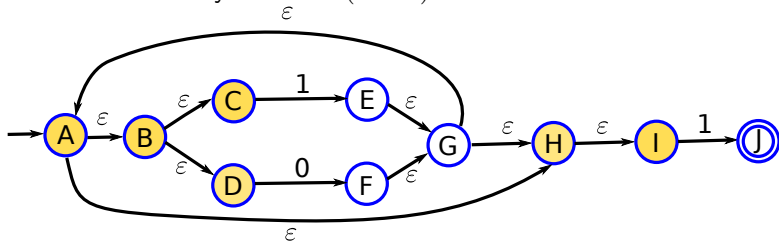
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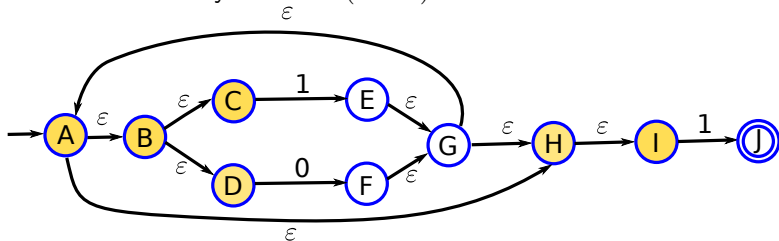
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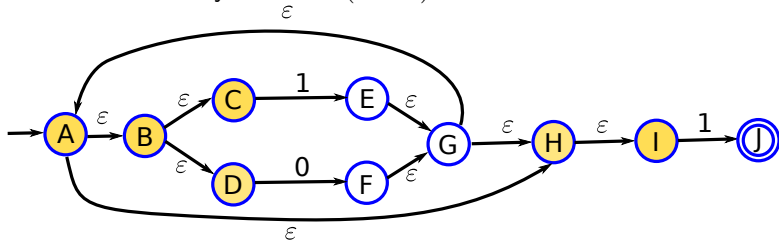
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→ ABCDHI

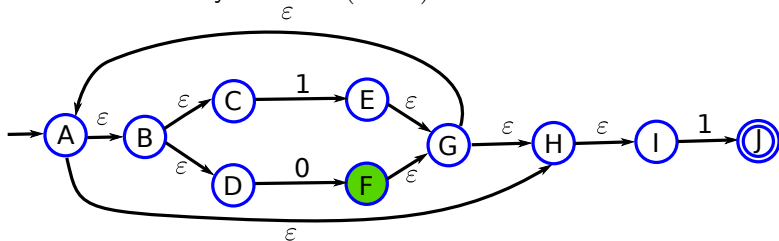
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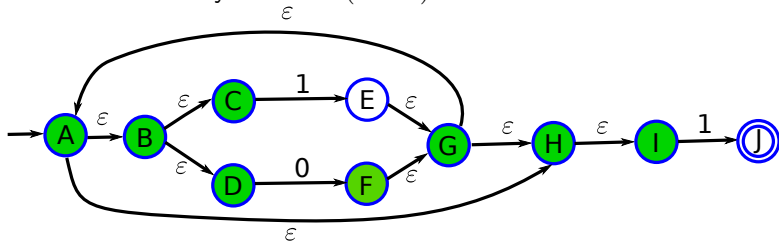
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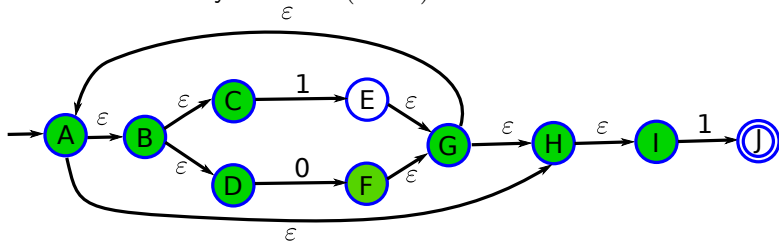
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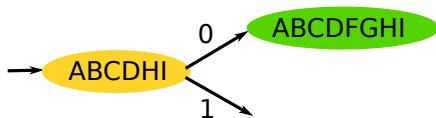
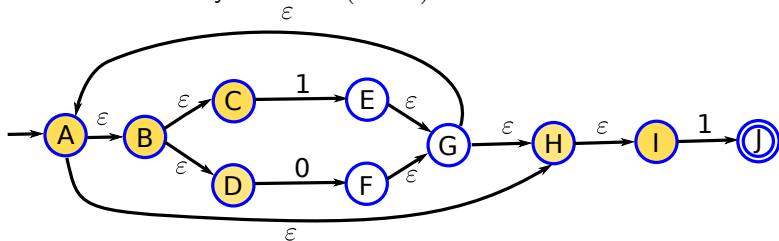
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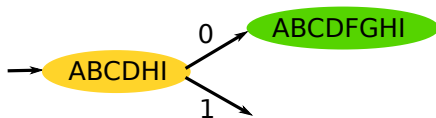
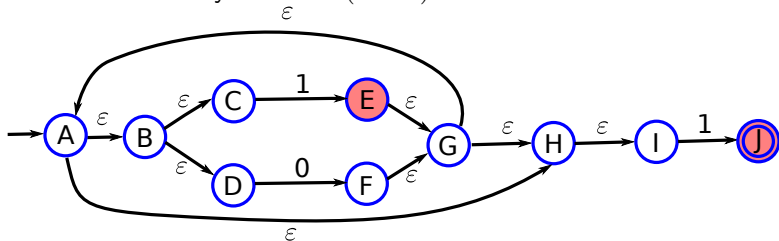
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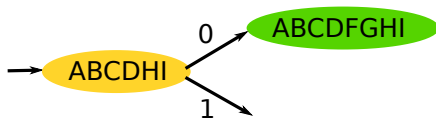
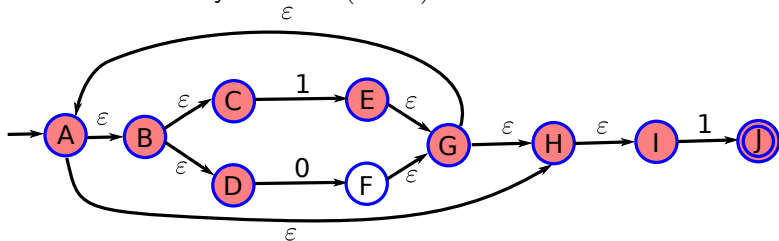
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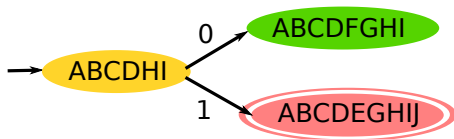
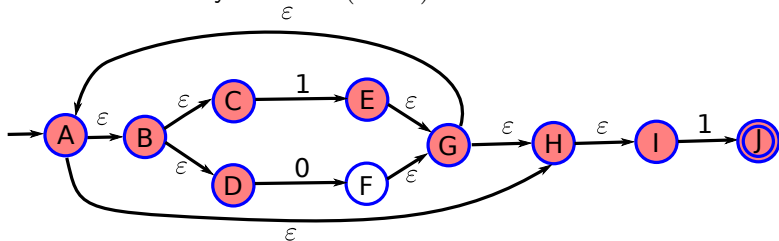
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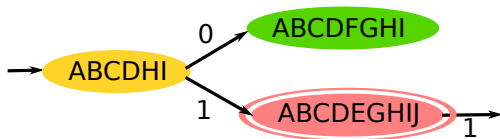
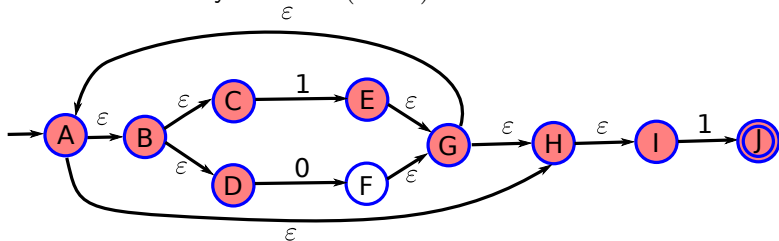
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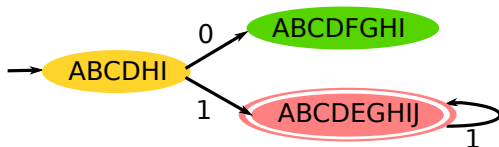
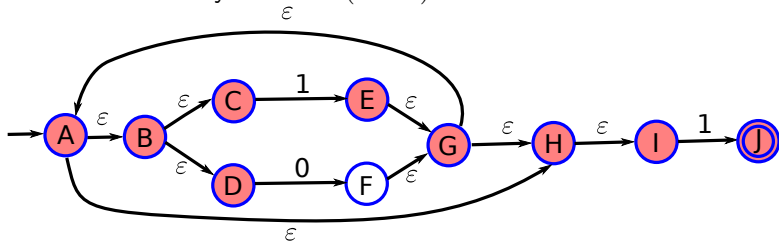
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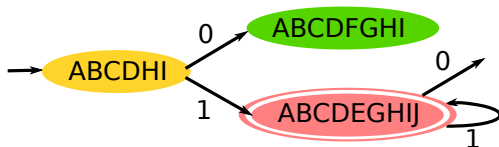
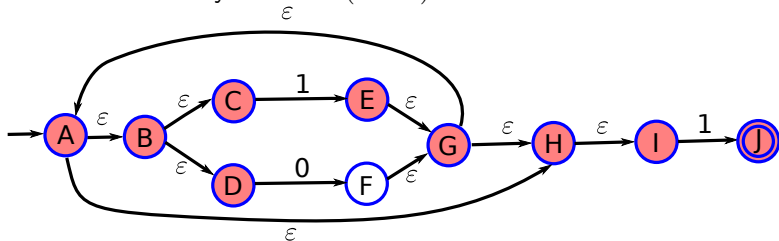
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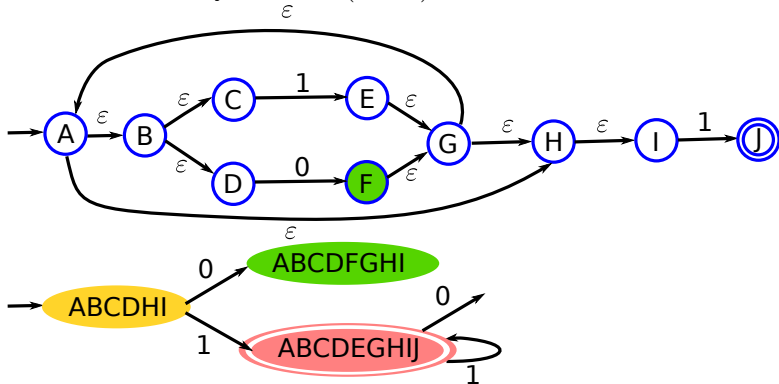
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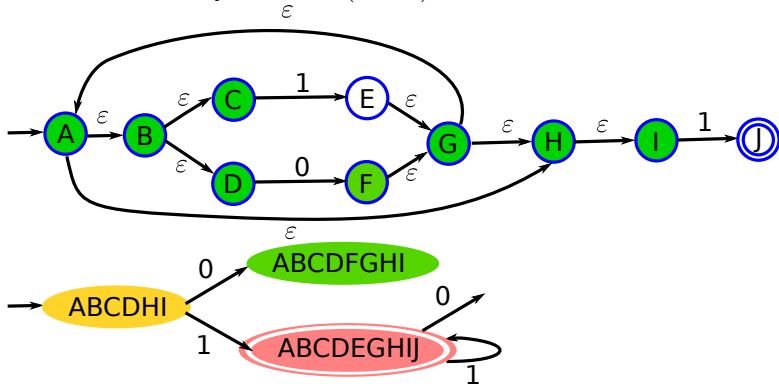
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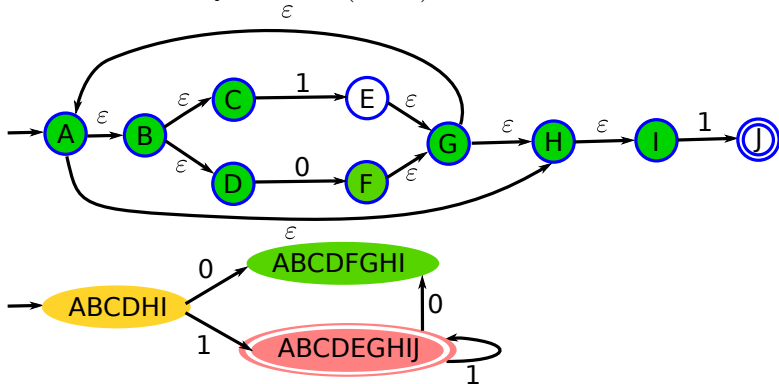
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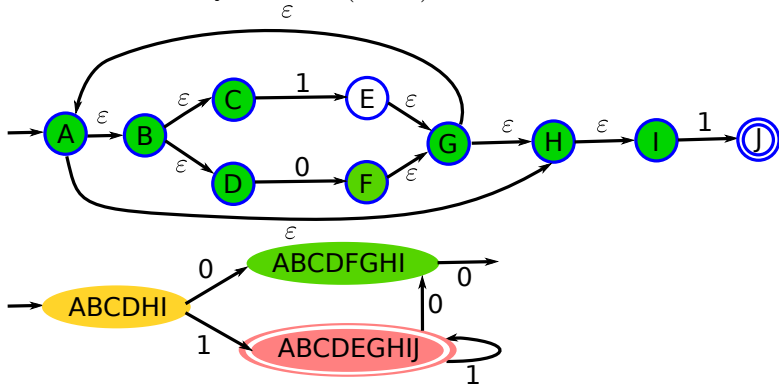
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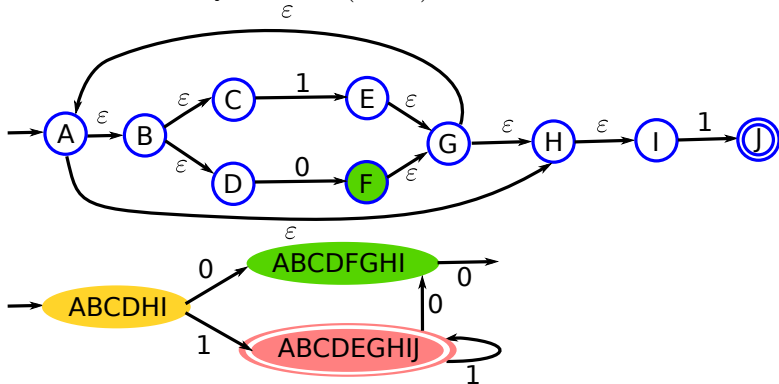
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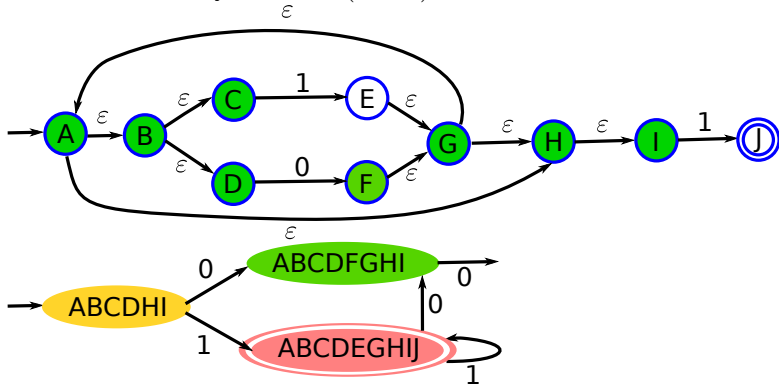
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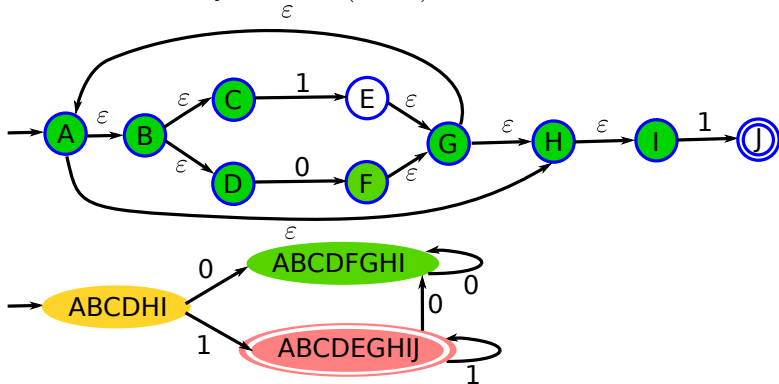
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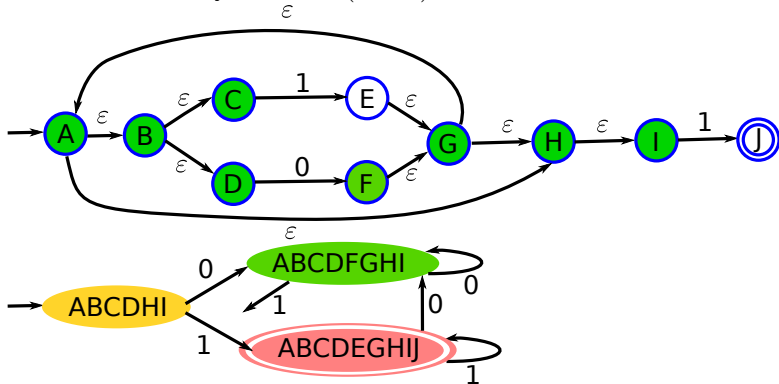
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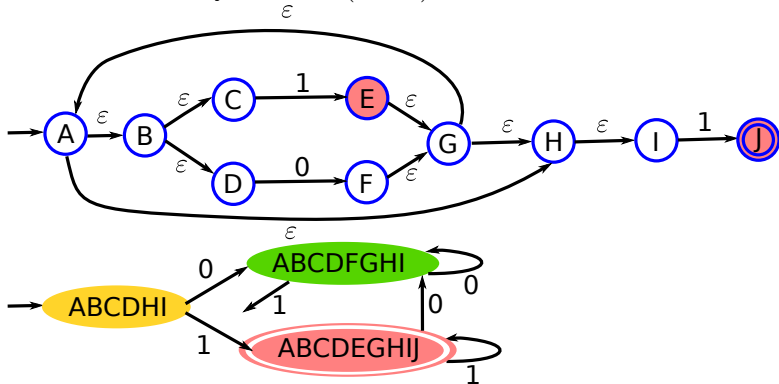
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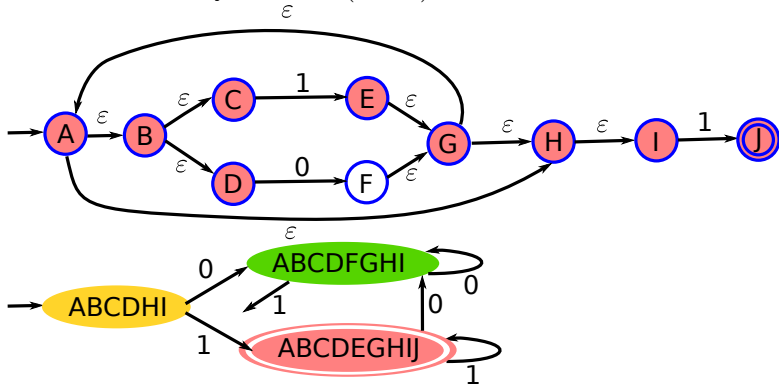
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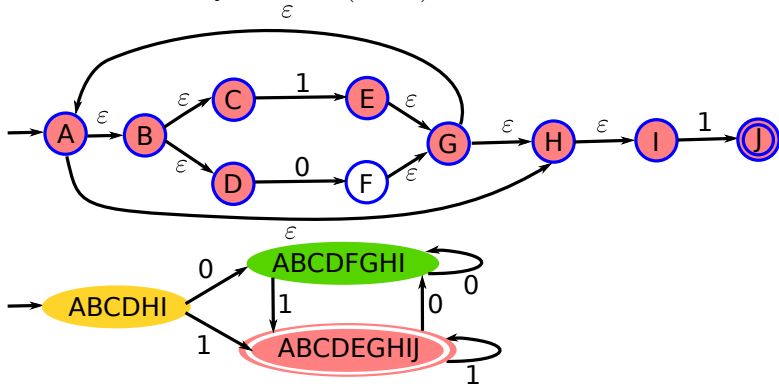
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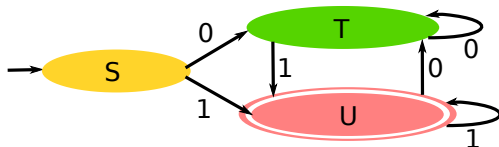
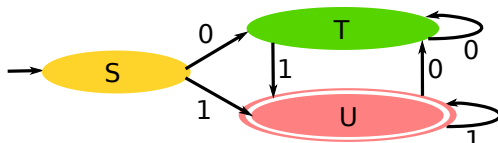


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	0	1
S	T	U
T	T	U
U	T	U

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- ▶ In practice, `flex`-like tools trade off speed for space in the choice of NFA and DFA representations