CS345H: Programming Languages

Lecture 5: Introduction to Parsing

Thomas Dillig

Outline

- Limitations of Regular Languages

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- Parser Overview

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- Context-free Grammars (CFGs)

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- Derivations

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- Ambiguity

Regular Languages

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- But regular languages are not expressive enough to turn a stream of tokens into structure
- For this, we need a more expressive formal language

Beyond Regular Languages

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- Many languages are not regular
- Classic Example: Strings of balanced parenthesis:

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- Question: Why is there no automata that can recognize this language?

What Can Regular Languages Express?

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- Languages requiring counting modulo a fixed integer
- Intuition: A finite automaton that runs long enough must repeat states
- Finite automaton cannot remember the number of times it has visited a particular state

Side Note: Comments in L

- Recall: Comments in L start with `(*`, end with `*)` and can be nested

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- Also Recall: Comments are removed during lexing
- Question: Are comments in L a regular language?

The Functionality of the Parser

- Input: sequence of tokens from the lexer

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- Input: sequence of tokens from the lexer
- Output: parse tree of the program

Example

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  `if x<>y then 1 else 2`

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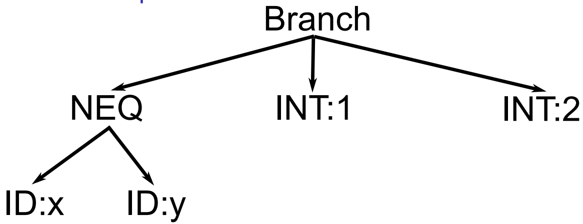
- Parser Output:



Parsing vs. Lexing

Phase	Input	Output
Lexer		
Parser		

Parsing vs. Lexing

Phase	Input	Output
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- We need:
  - A language for describing valid strings of tokens
  - A method for recognizing if a string of tokens is in this language or not

Context-free Grammars (CFGs)

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- Programming language constructs have recursive structure
- Example: An L expression is `expression + expression`, `if expression then expression else expression`, ...
- Context free grammars are a natural notation for this recursive structure

CFGs in more detail

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- A CFG consists of:
  - A set of terminals T
  - A set of non-terminals N
  - A start symbol S (non-terminal)
  - A set of productions

$$X \rightarrow Y_1 Y_2 \ldots Y_n$$

where $X \in N$ and $Y_i \in (T \cup N \cup \{\varepsilon\})$

Notational Conventions in this Class

- Non-terminals are always written upper-case

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- Non-terminals are always written upper-case
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- The start symbol is the left-hand side of the first production

CFG Examples

- A fragment of L

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```
EXPR → if EXPR then EXPR else EXPR
        |  EXPR + EXPR
        |  id
```

CFG Examples continued

- Simple arithmetic expressions:

CFG Examples continued

- Simple arithmetic expressions:

```
EXPR → E * E
        | E + E
        | (E)
        | id
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  3. Repeat (2) until there are no non-terminals in the string

The Language of a CFG continued

- More formally, write

$$X_1 \dots X_i \dots X_n \rightarrow X_1 \dots X_{i-1} \, Y_1 \dots Y_m X_{i+1} \dots X_n$$

if there is a production

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  $$X_i \rightarrow Y_1 \ldots Y_m$$

- Abbreviation: Write $X_1 \ldots X_n \rightarrow^* Y_1 \ldots Y_m$ if $X_1 \ldots X_n \rightarrow \ldots \rightarrow Y_1 \ldots Y_m$ in 0 or more steps

The Language of a CFG continued

- Now, let G be a context-free grammar with start symbol S. Then the language of G is:

$$\{a_1 \ldots a_n | S \rightarrow^* a_1 \ldots a_n \text{ and every } a_i \text{ is a terminal}\}$$

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- Once generated, terminals are permanent
- Question: What should terminals be when parsing a programming language?
- Answer: Tokens

Examples

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- CFG:

$$S \rightarrow (S)$$
$$S \rightarrow \varepsilon$$

Examples

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- Strings of balanced parentheses:

$$\{(^i)^j | i \geq 0\}$$

- CFG:

$$S \rightarrow (S)$$
$$S \rightarrow \varepsilon$$

or equivalently

$$S \rightarrow (S) \mid \varepsilon$$

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- Some strings in this language:

- ```
  ID
  IF ID THEN ID ELSE ID
  ID + ID
  IF ID THEN ID+ID ELSE ID
  IF IF ID THEN ID ELSE IF THEN ID ELSE ID
  ...
  ```
  ```

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EXPR → E * E
         |  E + E
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```

- Some strings in this language:

- `id`
  `(id)`
  `(id)*id`
  `id+id`
  `id*id`
  `id*(id) ...`

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- The idea of a CFG is a big step towards parsing tokens.
- But we don't just want to know if a string of tokens is in a language, we also need parse tree of input tokens
- Must also handle errors gracefully
- Need an implementation of CFGs (e.g., bison)

From Derivations to Parse Trees

- A derivation is a sequence of productions

$$S \to \ldots \to \ldots \to \ldots$$

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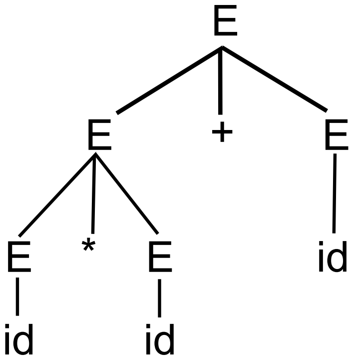
- A derivation can be drawn as a tree
  - Start symbol is the tree's root
  - For a production $X \to Y_1 \ldots Y_n$ add children $Y_1 \ldots Y_n$ to node X

Derivation Example

```
  E
→ E+E
→ E*E+E
→ id*E+E
→ id*id + E
→ id*id + id
```



Derivation in Detail

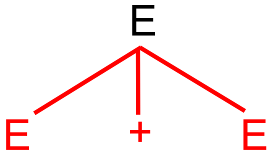
E

E

Derivation in Detail

E

$\rightarrow$ E+E



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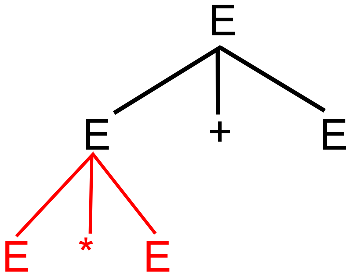
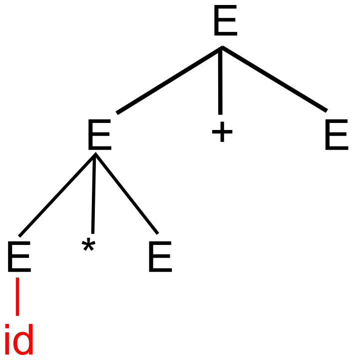


Derivation in Detail

$$\begin{array}{ll} & \texttt{E} \\ \rightarrow & \texttt{E+E} \\ \rightarrow & \texttt{E*E+E} \\ \rightarrow & \texttt{id*E+E} \end{array}$$



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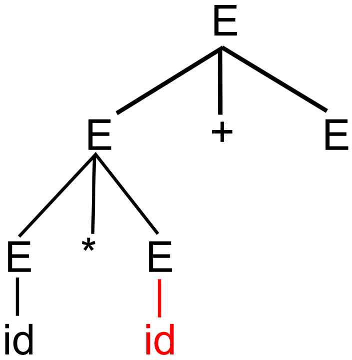

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→ id*id + id
```

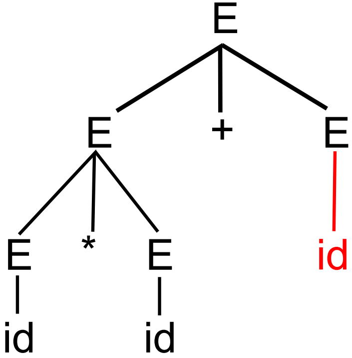

Notes on Derivations

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- The parse tree shows the associativity of operations, the input token string does not
- Example: The parse tree from the last slide encodes that times has higher precedence than plus

Left-most and Right-most Derivations

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- This means: At each step, we replace the left-most non-terminal
- There is also an equivalent notion of right-most derivation

Right-most Derivation in Detail

E

`E`

Right-most Derivation in Detail

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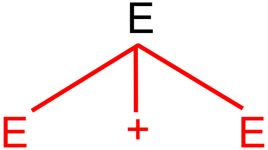
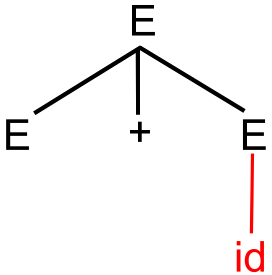
$\rightarrow$ E+E



Right-most Derivation in Detail

E
$\rightarrow$ E+E
$\rightarrow$ E+id



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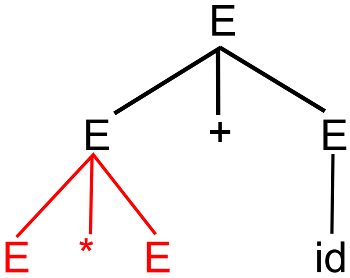

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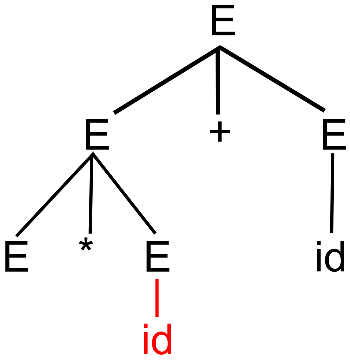

Right-most Derivation in Detail

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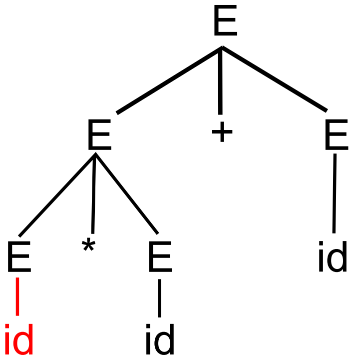

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- Observe that left-most and right-most derivations have the same parse tree
- The only difference is the order in which branches are added
- But when parsing tokens, we only care about the final parse tree, which may have many different derivations
- Left-most and right-most derivations are important in parser implementations

Ambiguity

- Recall our example grammar:

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       | (E)
       | id
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- Now, consider the string `id*id+id`

Ambiguity continued

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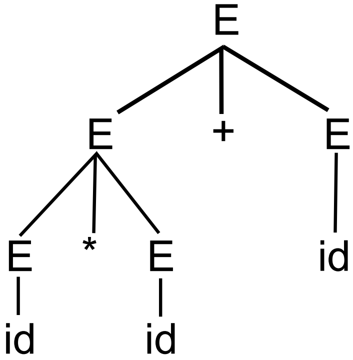

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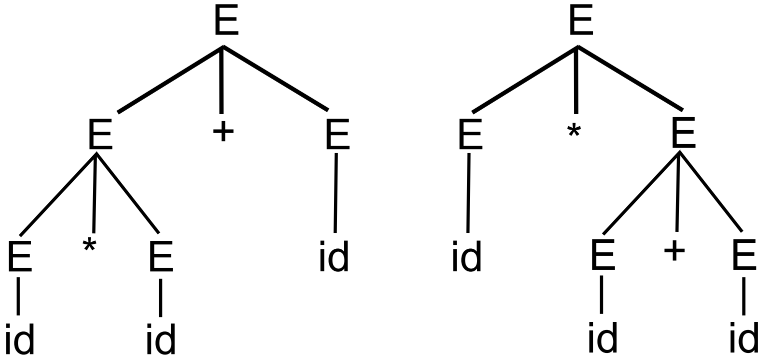

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- Ambiguity is bad!
- Leaves meaning of programs ill-defined

Dealing with Ambiguity

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$$
\begin{array}{lcl}
S & \rightarrow & E + S \mid E \\
E & \rightarrow & \mathsf{id} * E \mid \mathsf{id} \mid (S) * E \mid (S)
\end{array}
$$

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- It is also fundamentally impossible to transform an ambiguous grammar into a unambiguous grammar
- For this reason, tools such as `bison` include disambiguation mechanisms

Precedence and Associativity

- Instead of rewriting the grammar:

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- The parser tool `bison` allows you to declare precedence and associativity for this

Associativity Declarations

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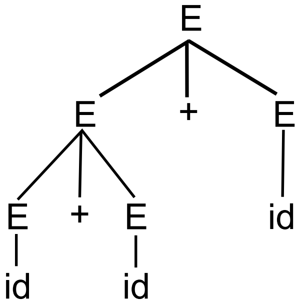

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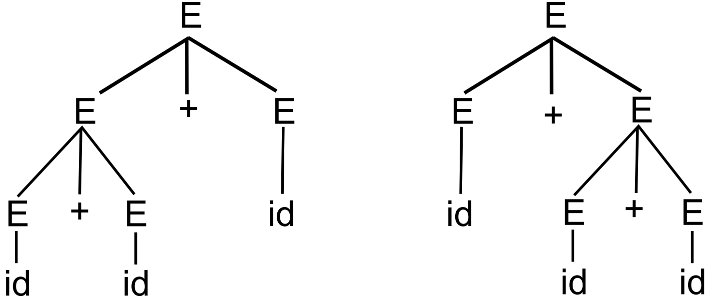

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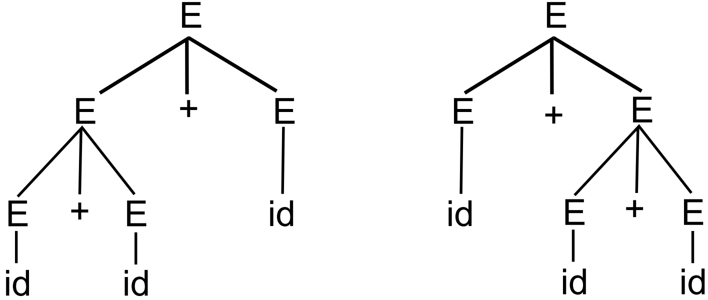

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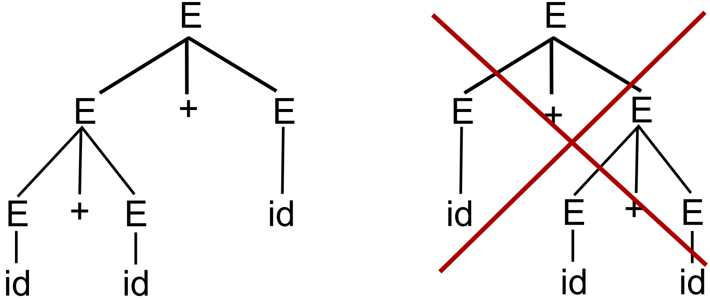

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Precedence Declarations

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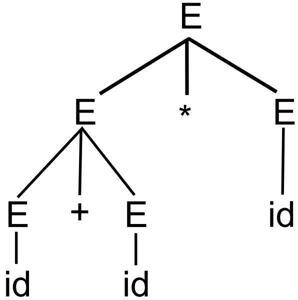

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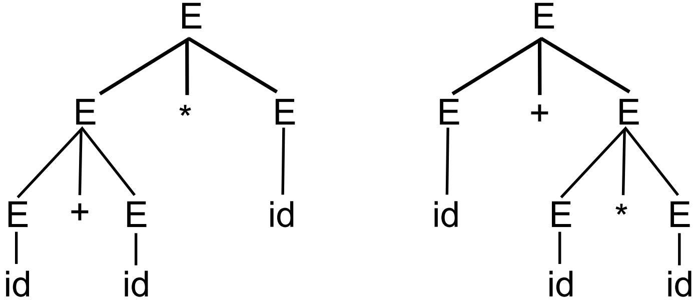

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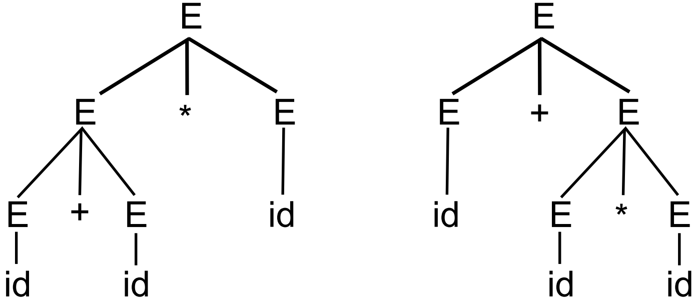

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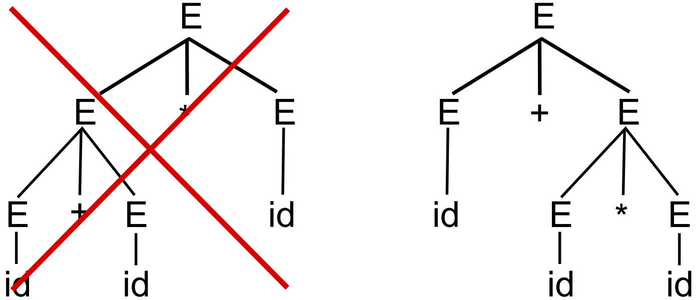

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