CS345H Midterm 1
Fall 2015/2016

- Please read all instructions (including these) carefully.
- There are 5 questions on the exam, all with multiple parts. You have 75 minutes to work on the exam.
- The exam is closed book, closed notes, closed computers, phones, tablets, etc.
- Please write your answers in the space provided on the exam, and clearly mark your solutions. Please do not use any additional scratch paper.
- Solutions will be graded on correctness and clarity. Each problem has a relatively simple and straightforward solution. You may get as few as 0 points for a question if your solution is far more complicated than necessary. Partial solutions will be graded for partial credit.

NAME: ____

UT email address: __________________________________

EID: ____

In accordance with the letter and the spirit of the UT Austin honor code I have neither given nor received assistance on this examination.

SIGNATURE: ____

Problem	Max points	Points
1	20	
2	15	
3	20	
4	15	
5	10	
Total	80	

1. (20 points) Lambda Calculus

   (a) (5 points) Reduce the following lambda calculus expression as much as possible. If the expression does not converge, write "divergent".
   $(\lambda x.x\ x)(\lambda x.x\ x)$

   Solution: $(\lambda x.x\ x)(\lambda x.x\ x) \rightarrow^{\beta} (x\ x)[(\lambda x.x\ x)/x] \rightarrow^{\beta} (\lambda x.x\ x)(\lambda x.x\ x) \rightarrow^{\beta} \textit{divergent}$

   (b) (5 points) Reduce the following lambda calculus expression as much as possible. If the expression does not converge, write "divergent".
   $(\lambda b.b)((\lambda a.a)(\lambda b.a\ b))$

   Solution: $(\lambda b.b)((\lambda a.a)(\lambda b.a\ b)) \rightarrow^{\beta} (\lambda b.b)(\lambda b.a\ b) \rightarrow^{\beta} (\lambda b.a\ b)$

(c) (10 points) Is the following expression a fixed point operator in Lambda calculus?

$$Z = \lambda f.(\lambda x.xx)(\lambda x.f(xx))$$

Prove or disprove your answer.

Solution: Yes, this is a fixed point operator. To prove this, we need to show that $Z\ h = h(Z\ h)$ for any function h. $(\lambda f.(\lambda x.xx)(\lambda x.f(xx)))h \rightarrow (\lambda x.xx)(\lambda x.h(xx)) \rightarrow (\lambda x.h(xx))(\lambda x.h(xx)) \rightarrow h((\lambda x.h(xx))(\lambda x.h(xx)))$ Here, the last two expressions show that $Z\ h = h(Z\ h)$.

2. (15 points) Consider the following string over the alphabet $\Sigma = \{a, b, c\}$:
$abaaabbbcbaa$
Give a flex specification that tokenizes this string as follows containing no more than two rules with no more than 4 symbols each:

$ab|a|a|abbb|cb|a|a$

Solution:

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ab^* { }
cb   { }
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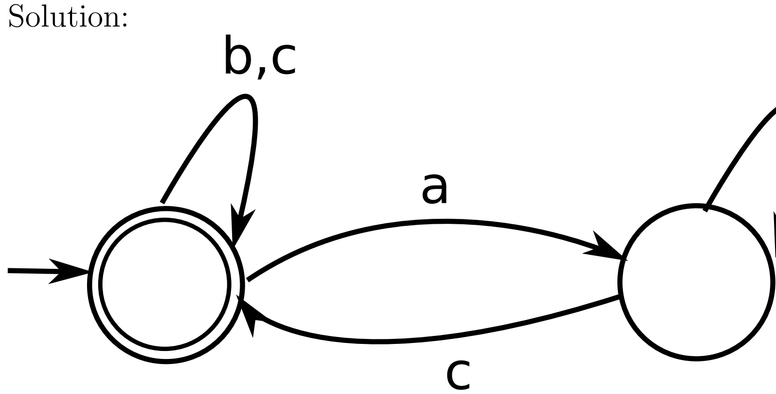
3. (20 points) Regular Languages

   (a) (7 points) Write a regular expression over the alphabet $\Sigma = \{a, b, c\}$ for all strings such that every a is followed by at least one c before the next a. Do *not* use flex notation.

   Solution: $((ab^*c^+) + b)^*$

   (b) (7 points) Draw an NFA for the language from (a). Your NFA must have no more than four states.

   Solution:



(c) (6 points) Give a context-free grammar that generates the same language.

Solution:

$$\begin{array}{lcl} S & \rightarrow & \varepsilon \mid bS \mid cS \mid aS'cS \\ S' & \rightarrow & \varepsilon \mid S'b \end{array}$$

4. (15 points) Consider the following context free grammar describing strings in the alphabet $\Sigma = \{a, b, c\}$:

$$S \rightarrow \varepsilon \mid bS \mid cS \mid aScS$$

(a) (5 points) Give a one-sentence (no more that 20 words) description of the language described by this grammar.

Solution: Every a is followed by at least one c.

(b) (10 points) Is this language regular? If so, give a regular expression that generates the same language. If not, explain clearly why not.

Solution: This is not a regular language. It is possible to have an unbounded number of a's before any c, and to ensure that every a is matched by at least one c, we need to count this unbounded number of a's. This cannot be done by a DFA.

5. (10 points) Short answer questions:

   (a) (5 points) Is it possible to write a context-free grammar for every regular expression? Explain your answer.

   Solution: Of course. L(Regular) $\subseteq$ L(CFG)

   (b) (5 points) We saw in lecture that any DFA can be converted to an NFA that accepts the same language. Is it also possible to convert each NFA to a DFA? If so, explain how, if not, justify why not.

   Solution: Every DFA is also an NFA.